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Angular Spectrum of the Diffusion Neutron Flux

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ABSTRACT

The adaptation of the attenuation law of the narrow beam of particles to the attenuation of scattered diffusion flux is usually solved by using the empirical build-up factor. The main difference of the diffusion flux is the presence of a certain spectrum of angles under which the particles intersect the absorber. The process of forming a diffusion neutron flux before the intersection of the border with the absorber requires in-depth analysis.

The process of forming the diffusion neutron flux in material where going diffusion of neutrons and, in front of the boundary with the material where their absorption occurs are viewed. Was analyzed the impact of the distance from last scattering and angle of intersection the boundary.

The probability of boundary intersection the boundary was obtained and the angular spectrum of diffusion neutron flow was determined. The evolution of the angular spectrum when changing the thickness of diffusion layer was considered. The results obtained are the foundation for a further study of the attenuation of the diffusion neutron flow.

Keywords: diffusion neutron flux, attenuation, angular spectrum, mean angle

1 INTRODUCTION

The well-known attenuation law was obtained under experimental conditions for a narrow beam of particles. In practice, we often encounter a non-collimated flux of particles. To adapt the attenuation law to such conditions, it is usually to use a correction factor, the so-called build-up factor (BUF). It is believed that it can and should take into account the energy of the particles, the material and thickness of the shield and the shield geometry [1].

However, many years of experience indicate that the values of the BUF obtained in experiments by different authors may differ many times [2]. Such a state of study may indicate the incompleteness of the factors taken into account when determining the BUF. For example, when studying multilayer shielding [3, 4], it is indicated that BUFs is dependent on the shielding layers order. Therefore, it is important how the flux of particles in front of the shield is formed, and that flux can have difference in its characteristics, which affects further attenuation in the shield.

2 THE MODEL OF DIFFUSION NEUTRON FLUX

Let us consider the penetration of the formation of a diffuse neutron flux and its penetration through the flat boundary between the medium of the scatterer and the absorber.

Suppose that the scatterer is only able to scatter neutrons and does not absorb them, and the absorber does the opposite. This assumption is a simplification, since any material has a certain probability of both absorbing and scattering neutrons in dependence on its cross section of the corresponding reaction.

During scattering, an isotropic diffusion flux of thermal neutrons is formed, which corresponds to the same probability of scattering at any angle and uniformly distributed within the solid angle of 4π steradian. Note that only in half of the encounter is the direction of motion directed toward the infinite plane interface between the scatterer and absorber medium, as shown in Fig.1.

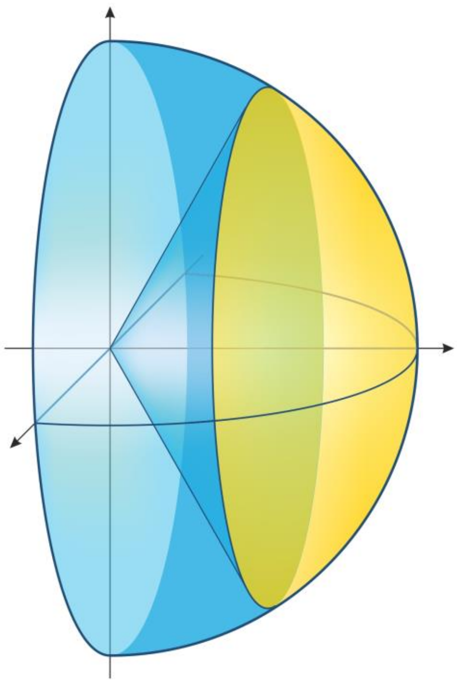

Figure 1: The 3D distribution of the scattering neutrons number

When the attenuation of a narrow beam of particles is considered, they are usually directed perpendicular to the outer surface of the absorber plate. In the case of a diffusion flux of neutrons, they cross the surface at different angles. Moreover, the distribution of the neutrons number will be uneven and such that the scattering frequency at the angle $\varphi = 0$ is the lowest, and at the angle $\varphi = \frac{\pi}{2}$ is the maximum, where φ - the deviation from the perpendicular to surface. The distribution of the number of neutrons $n_s(\varphi)$, which after scattering move at a certain angle is determined by a function proportional to the weighted circle of the radius $r = sin\,\varphi$, i.e.

$$n_s(\varphi) = \sin(\varphi) \tag{1}$$

Given that

$$\int_0^{\pi/2} n_s(\varphi) d\varphi = \int_0^{\pi/2} \sin(\varphi) d\varphi = 1 \tag{2}$$

the specified distribution is normalized to unity and can be used to average the diffusion flux characteristics depending on the scattering angle.

The average scattering angle is 60^0, that is, half of the number of neutrons is scattered within the cone with this angle and the other half outside this cone (see Fig. 1). The average scattering frequency in the range $0 \ldots \pi/2$ is $2/\pi = 0{,}63$ (see Fig. 2).

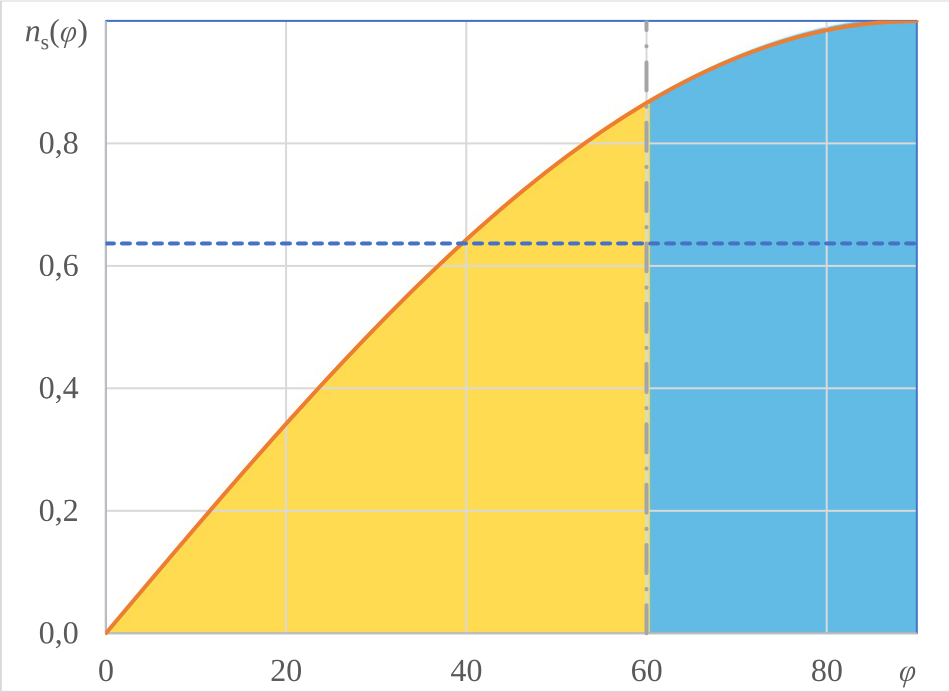

Figure 2: Distribution of the neutrons number by the scattering angle $n_s(\varphi)$

3 PROBABILITY OF CROSSING THE BOUNDARY

The diffusion flux that through the boundary is formed by neutrons that crossed the boundary after the last scattering and thus avoid the next scattering. Neutrons that have not crossed the boundary continue to move in the diffusion medium and can cross the boundary after further scattering.

Scattering occurs at a certain distance from the boundary plane, and only neutrons that overcome this distance will be able to cross the boundary. The probability of passing a distance x according to the known equation for a narrow beam, if the distance is substituted in relative units of mean free path (*mfp*)

$$p_1(x) = \exp(-x) \tag{3}$$

For the diffusion flux, there is also a dependence on the scattering angle, because the length of the path to the boundary depends on it, and therefore we have

$$p_2(x, \varphi) = \exp(-x/\cos(\varphi)) \tag{4}$$

At the scattering angle $\varphi > 0$, the distance that must be covered to reach the boundary increases, and at $\varphi = \frac{\pi}{2}$ it approaches infinity, because the neutron moves parallel to the boundary. In the limiting case $x = 0$, when the scattering occurs directly at the boundary, the probability does not depend on the scattering angle and all neutrons will cross the boundary with one hundred percent probability. However, as can be seen from Fig. 3, it seems that when $x > 0$ and $\varphi = \frac{\pi}{2}$ are small, the probability of overcoming the distance to the border immediately drops to zero.

It is important to note that the maximum number of neutrons is scattered at angle $\varphi = \frac{\pi}{2}$, and at the same time the probability of reaching the limit is zero. Conversely, the minimum amount is scattered at angle $\varphi = 0$, and at the same time the probability of crossing the border is maximum.

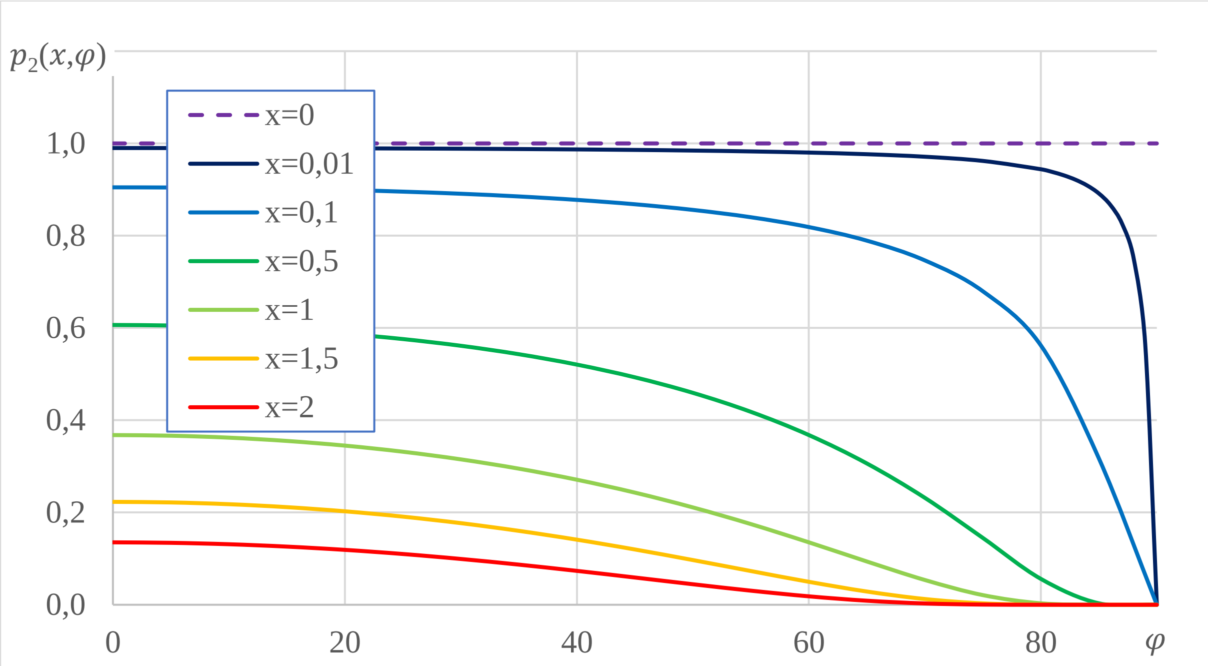

Figure 3: The probability of crossing the boundary $p_2(x,\varphi)$

Taking into account the distribution of the neutrons number from the scattering angle, the final probability of crossing the border at a certain angle is

$$p(x,\varphi) = n_s(\varphi)\, p_2(x,\varphi) = \sin(\varphi)\exp(-x/\cos(\varphi)) \quad (5)$$

The probability distribution presented in Fig. 4 at different distances from the scattering point to the boundary. The distribution maximum and, accordingly, the most probable angle of scattering φ_{max} at which the boundary will be crossed is depends significantly on the location of the scattering point. The values φ_{max} (see Table 1), calculated from the transcendental equation obtained under the condition that the derivative of $p(x,\varphi)$ is zero

$$x = \cos^3(\varphi_{max})\,/\sin^2(\varphi_{max}) \quad (6)$$

Table 1: The maximum probability of the crossing the boundary

x, mfp	0	0,001	0,01	0,1	0,5	1	1,5	2	2,5
φ_{max}, degree	90,0	84,3	77,8	64,3	48,9	41,0	36,3	32,9	30,4
p_{max}	1,0000	0,9851	0,9322	0,7155	0,3522	0,1744	0,0920	0,0502	0,0279

The distribution $p(x,\varphi)$ is uneven and asymmetric in the range of possible values of the scattering angle $\varphi = 0 \ldots \frac{\pi}{2}$ at all distances to the boundary. At small distances, the maximum probability shifted towards larger scattering angles. At relatively large distances $x > 1$, the maximum probability shifts towards small scattering angles. Apparently, when the maximum of the distribution (dotted line in Fig. 4) is near the middle of the range of scattering angle values, the distribution itself is significantly asymmetric, because for angles 75...900 the probability is very small, unlike in range $\varphi = 0 \ldots 15^0$. In addition, distributions normalized to the corresponding maximum value for a specific distance $p(x,\varphi)/(p\_max\,(x)\,)$ are presented (see Fig. 5), which visually confirms the specified evolution of the distribution.

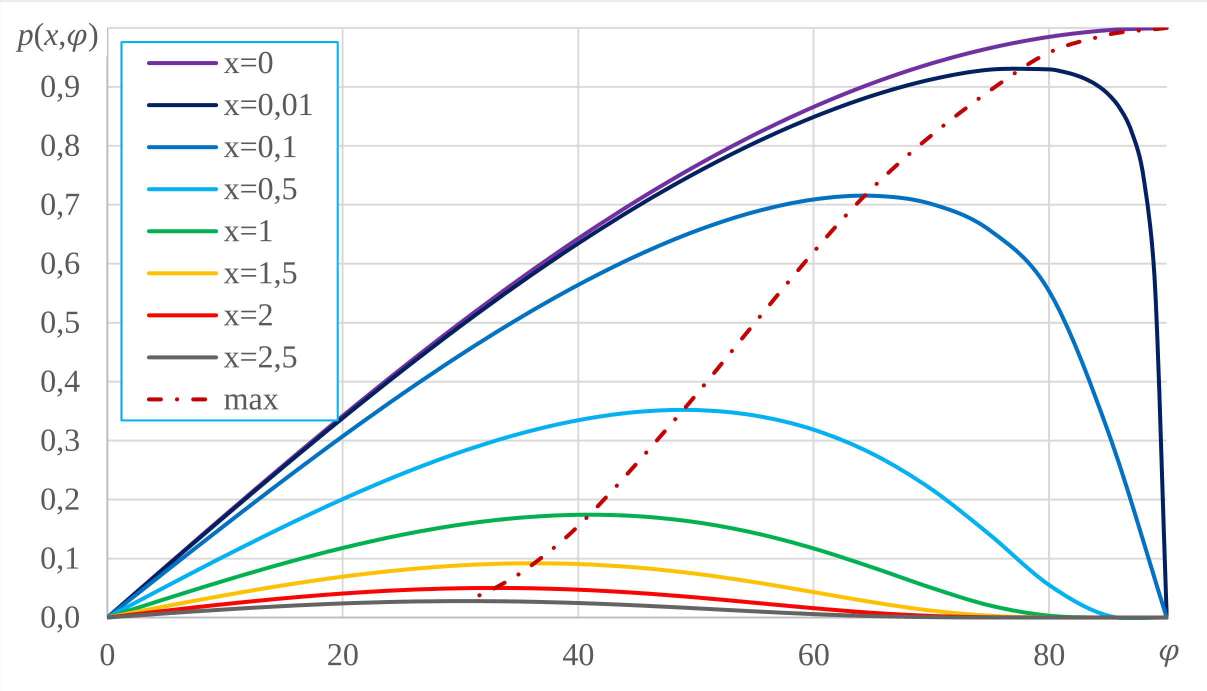

Figure 4: The probability of crossing the boundary $p_2(x,\varphi)$ with taking into account the number of neutrons

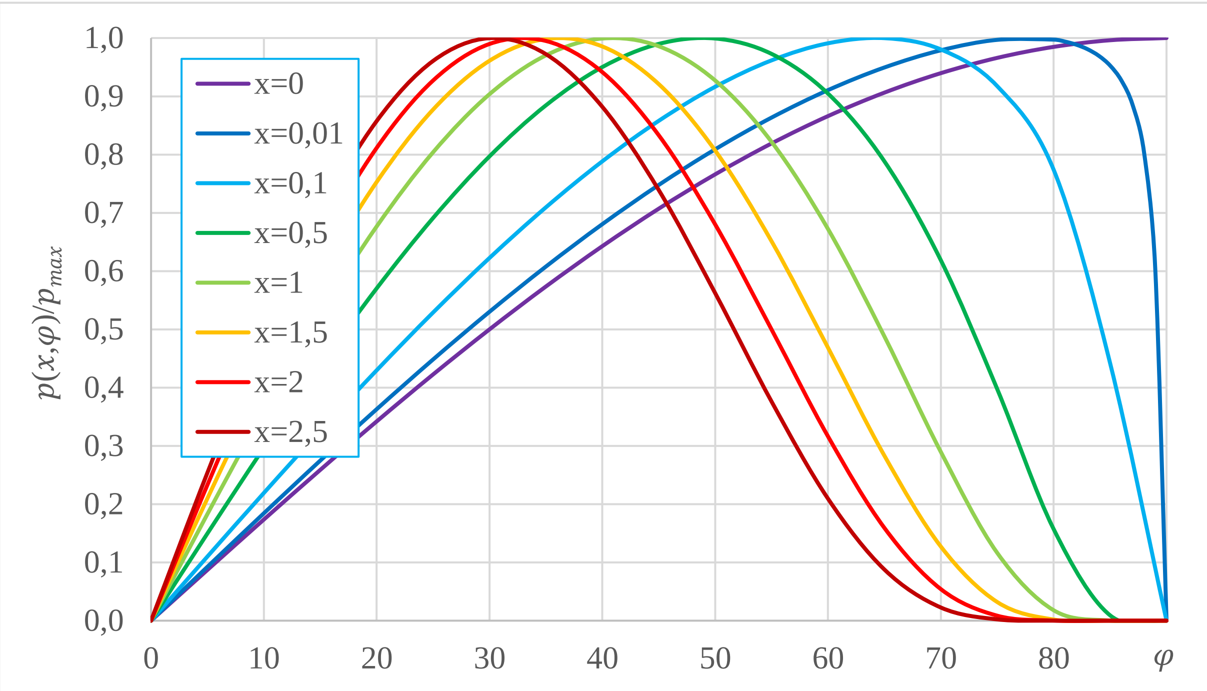

Figure 5: The distribution of the probability of crossing the boundary normalized to the maximum value $p(x,\varphi)/p_{max}(x)$

To estimate the dependence of the probability only on the distance to the boundary, we use the average integral value of the distribution, which characterizes the entire interval of possible values of the scattering angle and is calculated by the formula

$$\bar{p_s}(x) = \frac{1}{\pi/2}\int_0^{\pi/2} p(x,\varphi)d\varphi \quad (6)$$

The dependence of the probability of overcoming the distance to the boundary for the diffusion flux of neutrons is significantly different from the similar dependence for a narrow beam. For comparison, both dependencies presented in Fig. 6. In fact, the presented dependence gives the attenuation of the diffusion flux by scattering depending on the distance.

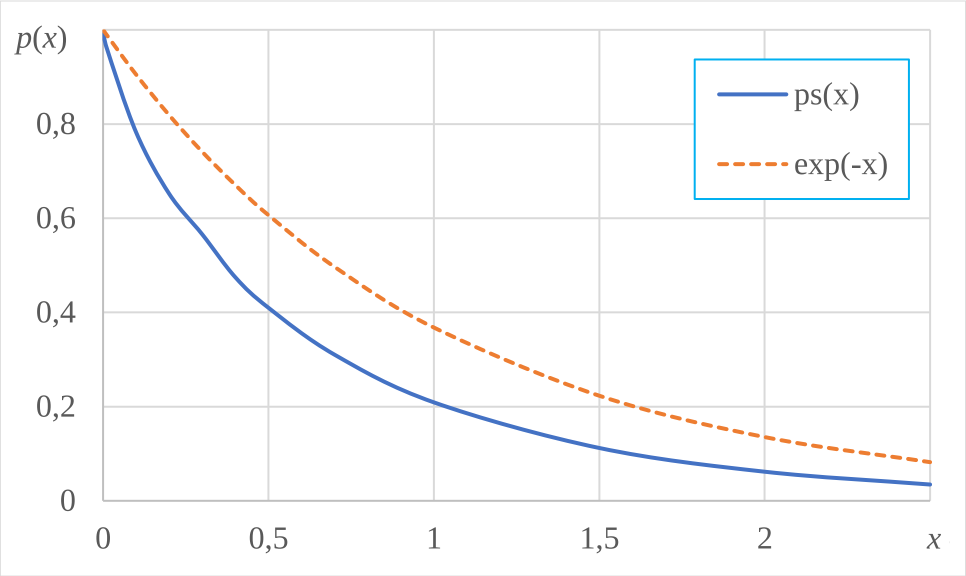

Figure 6: Attenuation of the diffusion flux due to scattering

The ratio of the attenuation values for the narrow beam and the diffusion flux of neutrons K is given in Table 2. As we can see, the attenuation of the diffusion flux is always greater than the attenuation of the narrow beam. Moreover, at short distances, they are initially quite close. Already at a distance of 0,1 mfp the difference between them exceeds 15%. At longer distances, the specified attenuation differs by several times. Ratio K is not constant and changes depending on the distance.

Table 2: The comparison of attenuation for the narrow beam and diffusion flux

x, mfp	0,001	0,01	0,1	0,5	1	1,5	2	2,5
$\overline{p}_s(x)$	0,995	0,964	0,782	0,410	0,209	0,112	0,062	0,035
$exp(-x)$	0,990	0,990	0,905	0,607	0,368	0,223	0,135	0,082
K	1,004	1,027	1,157	1,480	1,760	1,989	2,189	2,370

It is important to note that the dependence of the attenuation of the diffusion flux on the distance differs from the exponential dependence with a constant coefficient. That is, we can confidently state that the classical attenuation law, which usually used for a narrow beam, not applied to a diffusion flux. Adding a correcting constant factor such as the build-up factor BUF cannot solve the problem.

4 THE ANGULAR SPECTRUM OF THE DIFFUSION FLUX

We are currently considering attenuation in a medium where only scattering is possible, and after that the neutron does not die, but only changes its direction of motion until it crosses the boundary with the absorber. The weakening of the diffusion flux in the medium of the absorber requires a separate study, and for this, it is necessary to know the distribution by the scattering angle of the diffusion flux of neutrons that crosses the boundary between the diffuser and the absorber. This distribution of the number of neutrons $n(\varphi)$ will be called the spectrum by the scattering angle, or the angular spectrum of the neutron flux.

Further research aimed at obtaining a generalized spectrum of the diffusion flux by the scattering angle at the entrance to the absorber medium. If we sum up the scattering from all possible distances to the boundary in the interval 0...∞, which end with crossing the boundary at a certain scattering angle, we get

$$p_\infty(\varphi) = \int_0^\infty p_2(x,\varphi)dx = \int_0^\infty exp(-x/cos(\varphi))dx = cos(\varphi) \tag{6}$$

It is important to note that such integration based on the assumption that the amount of scattering is the same at all distances from the boundary, i.e., the flux of neutrons in the entire space in front of the boundary is the same. It previously shown that under certain conditions near the boundary the diffusion flux decreases by more than two times [5]. However, further calculations made based on these assumptions, and therefore the results that obtained should be considered as a limit estimate, or a certain iteration.

The cumulative distribution of the probability of crossing the boundary from all distances (6) has an expected character. Indeed, the greatest probability of crossing the border is at the scattering angle $\varphi = 0$, because in this case the path to the boundary will be the shortest from any distance x.

It is possible to obtain the spectrum of the diffusion flux of neutrons crossing the boundary by the scattering angle taking into account the distribution $n_s(\varphi)$ equation (1)

$$n(\varphi) = n_s(\varphi)\, p_\infty(\varphi) = \sin(\varphi)\cos(\varphi) = \frac{1}{2}\sin(2\varphi) \tag{7}$$

The distribution normalized per unit

$$\dot{n}(\varphi) = \sin(2\varphi) \qquad \int_0^{\pi/2} \dot{n}(\varphi)d\varphi = 1 \tag{8}$$

The scattering layer near the boundary has a significant contribution to the formation of the spectrum of the diffuse neutron flux. Therefore, it will be useful to estimate the contribution of different layers in front of the boundary. To investigate this, let us consider the option when scattering occurs only in the boundary layer of thickness h. The rest of the space in front of the boundary is a vacuum where scattering does not occur. Then

$$\begin{aligned} p_h(\varphi) &= \int_0^h p_2(x,\varphi)dx = \int_0^h \exp(-x/\cos(\varphi))dx = \\ &= -cos(\varphi)\, exp\left(-\frac{x}{cos(\varphi)}\right)\Big|_0^h \\ &= cos(\varphi)\left[1 - exp\left(-\frac{h}{cos(\varphi)}\right)\right] \end{aligned} \tag{9}$$

and the spectrum of the diffusion flux of neutrons by the scattering angle crossing the boundary formed due to scattering in a layer of thickness h normalized to unity is:

$$n_h(\varphi) = n_s(\varphi)\, p_h(\varphi) = \sin(2\varphi)\left[1 - \exp\left(-\frac{h}{\cos(\varphi)}\right)\right] \tag{10}$$

The dependence for a certain values of the scattering layer thickness shown in Fig. 7. As already mentioned above, the layers closest to the boundary make a significant contribution to the final spectrum in terms of the scattering angle. The spectrum at a layer thickness of $h = 3$ almost completely coincides with the spectrum at an infinite layer thickness. Neutrons arriving from the scattering layer $h = 0{,}5$ make up approximately half of all neutrons crossing the boundary.

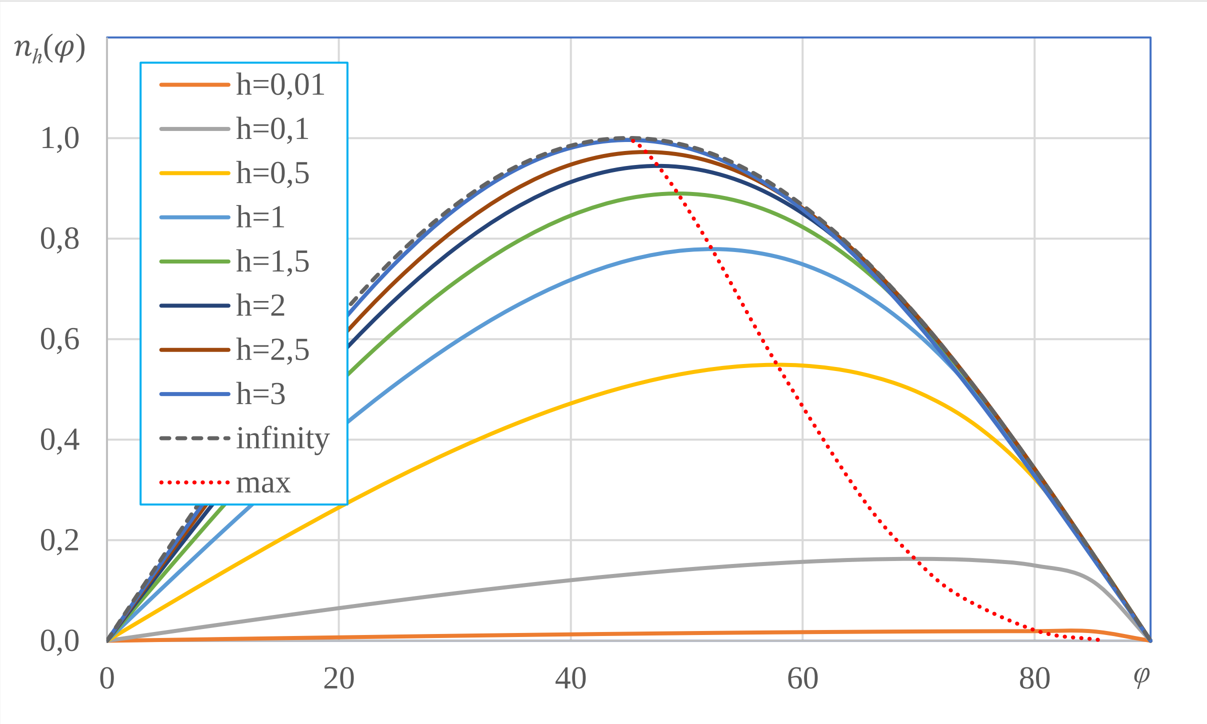

Figure 7: The angular spectrum of the diffusion flux at different thicknesses of the scattering layer

It is important to note, that the most likely scattering angle depends on the thickness of the layer that forms the diffusion flux, and can be found from the equation

$$\frac{d}{d\varphi}p_h(\varphi) = 0 \tag{11}$$
$$\frac{d}{d\varphi}\sin(2\varphi)\left[1 - \exp\left(-\frac{h}{\cos(\varphi)}\right)\right] = 0$$
$$2cos(2\varphi) - \left[2cos(2\varphi) - h\,sin(2\varphi)\frac{\sin(\varphi)}{\cos^2(\varphi)}\right]\exp\left(-\frac{h}{\cos(\varphi)}\right) = 0$$

Equation (11) is transcendental and cannot be expressed with respect to the scattering angle, but the solution can still be found through iterations. The results of the calculation of the most probable scattering angle $\varphi_{\max}$ at different values of the thickness of the scattering layer are given in Table 3 and shown in Fig. 5 as a dotted line.

Table 3: The most probable and average scattering angle of the diffusion flux

h	0	0,01	0,1	0,5	1	2	3	∞
$\varphi_{\max}$	90	80,3	69,7	57,8	52,2	47,5	45,9	45,0
$\bar{\varphi}$	60	56,6	54,2	50,0	47,7	45,9	45,3	45,0

As we can see, the accumulated spectrum from the scattering angle at infinite scattering layer thickness is symmetric with a maximum at $\varphi = 45^0$. The smaller the thickness of the scattering layer, the larger the maximum of the spectrum, up to $\varphi_{\max} = 90^0$. At $h = 0$, the spectrum coincides with the distribution of the number of neutrons $n_s(\varphi)$, because in this case all neutrons are guaranteed to cross the boundary.

The spectrum by the scattering angle and the degree of scattering of the diffusion flux formed in the scattering layer significantly affect the further attenuation of the flux in the absorber plate. Therefore, it is important to have a certain quantitative indicator of the diffusion flux spectrum, as measure of difference to a narrow beam. For example, you can use the average integral scattering angle for this

$$\bar{\varphi}(h) = \int_0^{\pi/2} \varphi \, n_h(\varphi) d\varphi \, / \int_0^{\pi/2} n_h(\varphi) d\varphi \qquad (12)$$

Values of the average scattering angle calculated by formula (12) are presented in Table. 2. As we can see, they change in the interval 45...600. The largest changes in $\bar{\varphi}(h)$ per unit of thickness occur at small thicknesses of the scattering layer. The greater the scattering angle at which the neutron crosses the boundary and enters the absorber medium, the greater the attenuation of the diffusion flux can be expected. In this way, the idea may arise that one should strive for a thin scattering layer, because it forms a better spectrum of the diffusion flux from the point of view of flux attenuation. However, it should also be taken into account that if the thickness of the scattering layer is too thin, the probability that scattering in it will not occur at all, that is, the neutron will slip through it, increases. Thus, the thickness of the scattering layer before attenuation in the absorber can be optimized and this requires a separate study.

5 CONCLUSION

1. The formation of the diffusion flux in the scattering layer is considered. An estimate of the probability of crossing the flat boundary with the absorber after passing through the scattering layer was obtained.
2. The attenuation of the diffusion flux in scattering layer differs from the exponent with a constant coefficient.
3. The angular spectrum of the diffusion flux crossing the boundary between the scatterer and the absorber depends significantly on the thickness of the scattering layer. With a reduced thickness of the scattering layer, the most likely scattering angle changes from 45^0 to 90^0.
4. The presence of a scattering layer before reaching the absorber layer can have a decisive effect on the attenuation of the flux. However, the diffusion flux through the boundary mainly consists of neutrons, the last scattering of which occurred at a distance of no more than 3 mfp to the boundary.

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