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ANALYTICAL MODEL OF HEAT TRANSFER IN FUEL ROD SUITABLE FOR NEUTRON CALCULATIONS

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ABSTRACT

In the domain of nuclear engineering, the precise understanding of a reactor core's response is paramount for ensuring its safe and efficient operation. This paper introduces a preliminary work on novel analytical model for heat transfer within a fuel rod, a critical component in nuclear reactors. By employing simplified one-dimensional radial geometry and assuming constant material properties along concentric rings, this model strikes a balance between computational efficiency and accuracy. It addresses heat transfer in the fuel pellets, cladding, and the fuel-cladding gap, with an initial focus on an open gap scenario filled with inert helium gas and fresh fuel, i.e., zero burnup. The model incorporates complex considerations, such as gas conductance, radiative heat transfer, and shedding light on the intricate interplay of these factors. Through rigorous mathematical derivations, the paper provides solutions for temperature distributions in the fuel and cladding, elucidating their dependencies on key parameters. This work offers a tool for estimating heat transfer in fuel rods, facilitating essential neutron calculations crucial for reactor core safety and efficiency. The paper also explores the mechanical aspects of the gap, including gap thickness and gas inventory, as well as thermal aspects, such as heat transfer calculations across the gap. This work provides a simplified yet time-efficient tool for estimating heat transfer within fuel rods, thereby aiding in neutron calculations that are vital for ensuring the safety and efficiency of reactor cores.

1 INTRODUCTION

In the field of nuclear engineering, understanding the response of a reactor core is crucial for ensuring safe and efficient operation. Key to this understanding, are the thermohydraulic parameters of the fuel and the distribution of the neutron flux or thermal power, as these two sets of data are intricately related. They are approximately proportional under certain conditions, which is also assumed in this work. Neutron transport calculations, which determine the power distribution within the core, rely on temperature distributions in the fuel pellets, fuel rod cladding, and the moderator coolant. These temperature distributions, in turn, result from thermohydraulic calculations based on thermal power distributions, which is obtained by neutron transport calculations. However, accurately determining the temperature distribution within the fuel rod presents a complex challenge due to the changing material properties of the fuel during combustion and the incomplete knowledge of all the mechanisms involved.

This paper aims to propose preliminary study on the development of a simplified yet time-efficient analytical model of heat transfer in a fuel rod for neutron transport calculations. Initial approach employs one dimensional radial geometry, where material properties remain constant along the concentric rings. By assuming this simplification, we aim to achieve the necessary accuracy while minimizing computational complexity. The model proposes heat transfer in (i) fuel pellet, (ii) cladding and (iii) pellet-cladding gap. The gap conductance is calculated using two summed heat paths: filled gas conductance and radiative heat transfer (initially considering open gap accounting for zero burnup with filled inert gas helium).

2 HEAT TRANSFER IN FUEL PELLET

The general heat transfer equation in cylindrical coordinates is given by,

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\cdot r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k\cdot r\frac{\partial T}{\partial \theta}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+q'''=\rho C_p\frac{\partial T}{\partial t} \tag{1}$$

Where

C_p = is the specific heat capacity

k = the thermal conductivity coefficient

q''' = the thermal power density.

For simplicity, we are considering the steady state heat transfer only in radial direction (no azimuthal or axial dependence), where the properties of material can be considered constant along concentric rings. Due to the typical cylindrical geometry of the fuel pins, the assumption of no azimuthal dependence is valid for most types of reactors. However, the assumption of no axial dependence fails at the top and at the bottom of the active fuel height, and for some specific reactor types (e.g., TRIGA) also broader [3]. These assumptions drop out various terms in Eq. 1 and simplify the equation in uni-dimensional steady state form.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k\cdot r\frac{\partial T}{\partial r}\right)+q'''=0 \tag{2}$$

Considering thermal conductivity *k* as constant and the general solution of the Eq. (2) is given by:

$$T(r)=-\frac{q'''r^2}{4k}+A\ln r+B \tag{3}$$

Using the thermal symmetry boundary conditions, $T(R_{fo})=T_{fo}$ and $\frac{dT}{dr}=0$ for $r=0$, the value of constants A and B in Eq. (3) are evaluated as,

$$A=0,\quad B=T_{fo}+\frac{q'''R_{fo}^2}{4k}$$

The resulting temperature distribution and the fuel centreline temperature is maximum at ($r = 0$) in the considered cylindrical fuel pellet will be:

$$T(r) = T_{fo} + \frac{q'''}{4k} R_{fo}^2 \left(1 - \frac{r^2}{R_{fo}^2}\right) \tag{4}$$

3 HEAT TRANSFER IN CLADDING

We considered steady state case with no volumetric heat generation rate ($q''' = 0$) and uniform thermal conductivity (k), the mono dimensional heat transfer equation Eq. (2) takes the following form,

$$\frac{k}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \tag{5}$$

General solution of the Eq. (5) is,

$$T(r) = C \ln r + D \tag{6}$$

Using the boundary conditions $T(T_{ci}) = T_{ci}\,, T(T_{co}) = T_{co}$, solving for C and D and substituting into the general solution, we obtain,

$$T(r) = \frac{T_{ci} - T_{co}}{\ln {}^{R_{ci}}/_{R_{co}}} \ln\left(\frac{r}{R_{co}}\right) + T_{co} \tag{7}$$

By Fourier law, the local heat flux density is:

$$q'' = -k\frac{dT}{dr} \quad => \quad \frac{q'}{2\pi r} = -k\frac{dT}{dr}$$

Separating the variable and integrating from R_{ci} to R_{co},

$$\int_{T_{co}}^{T_{ci}} dT = -\frac{q'}{2\pi k}\int_{R_{co}}^{R_{ci}} \frac{1}{r}\,dr$$

$$\Delta T = \frac{q'}{2\pi k} \ln\frac{R_{co}}{R_{ci}} \tag{8}$$

Thermal conductivity k of zircaloy with 1% Nb. is taken [4][3]:

$$k = 15.0636 \cdot e^{(0.4618\times 10^{-3}\cdot T)} \tag{9}$$

Since the cladding is not so thick, we can simplify considering the temperature profile to be linear by following assuming, $R_{co} \approx R_{ci} \cong R_c$, $R_{co} - R_{ci} = \delta_c => R_{co} = R_{ci} + \delta_c =>$ $R_{co} = R_c + \delta_c$

Therefore, the Eq. 8 takes the form as,

$$\Delta T = \frac{q'}{2\pi k}\ln\frac{R_{c+}\delta_c}{R_c} = \frac{q'}{2\pi k}\ln\left(1+\frac{\delta_c}{R_c}\right) \approx \frac{q'}{2\pi k}\left(\frac{\delta_c}{R_c}\right)$$

$$\Delta T = q'\frac{\delta_c}{2\pi k R_c} \quad (10)$$

Here, $\Delta T = T_{ci} - T_{co}$, and the cladding outer temperature is estimated using the coolant temperature using the equation of heat transfer by convection:

$$T_{co} = \frac{q'}{2\pi R_{co} h} + T_{fl}$$

4 HEAT TRANSFER IN THE GAP

The gap behaviour is quite complicated due to different underlying physical phenomena such as the evolution of gap due to expansion or contraction of the fuel and/or cladding, change in gap inventory due to fission gas release, single and multi-gas properties and heat transfer characteristics in gap. In the literature, the gap behaviour is categorized as: (i) the mechanical aspects of the gap (e.g. gap thickness, internal pressure, gas inventory); and (ii) the gas thermal aspects (e.g. heat transfer calculations across the gap)[1][2]. The gap conductance as three summed parallel heat paths is given by,

$$h_{gap} = h_g + h_r + h_c \quad (11)$$

where,

$h_g =$ filled gas conductance
$h_r =$ radiative heat transfer (direct thermal radiation)
$h_c =$ conductance because of contact

The heat transfer through the gap,

$$q'' = h_{gap}(T_{fo} - T_{ci})$$

$$\Delta T = \frac{q'}{2\pi h_{gap} R_g} \quad (12)$$

Heat transfer coefficient $h_g (W/m^2K)$ for filled gas is:

$$h_g = \frac{k_{gas}}{L} \quad (13)$$

Where k_{gas} is the conductance of gas. Generally, for the mixture of different types of gas the conductivity is calculated using $k_{gas}^{mix} = \prod_i k_i^{xi}$. In our study (zero burnup analysis) we considered only helium as filled gas and L is taken as effective gap length i.e., δ_{eff}, which is calculated as $\delta_{eff} = \delta_g + \delta_{jf} + \delta_{jc} = \delta_g + \delta_{jump}$ (where δ_{jf}, δ_{jc} and δ_g is the jump of fuel, jump of cladding and gap). Therefore, Eq. 13 take up the form as:

$$h_g = \frac{k_{gas}}{\delta_{eff}} = \frac{k_{gas}}{\delta_g + \delta_{jump}} \tag{14}$$

For k_{gas} (thermal conductivity of gas - $\frac{W}{mK}$) the correlation used is same as used in FINIX code [5], where *A* and *B* are constants and *T* is temperature in Kelvin i.e.,

$$k_{gas} = A\,T^B, \;\; A = 2.53 \times 10^{-3}\,, B = 0.7146$$

For the calculation of jump length (δ_{jump}) the correlation is taken from FRAPTRAN 1.5 code:

$$\delta_{jump} = \frac{a\left[k_{gas}\,{T_{gas}}^{0.5}/P_{gas}\right]}{\sum_j \left(f_j a_j / M_j^{0.5}\right)}$$

where:
$a = 0.024688$
T_{gas} = temperature of the gas in fuel cladding gap (K)
P_{gas} = pressure of the gas in fuel-cladding gap (N/m^2)
f_j = mole fractions of the j-th gas component
a_j = accommodation coefficient of the j-th gas component
M_j = molecular weight of the j-th gas component

The accommodation coefficient is taken by the equation used in FRAPTRAN 1.5 [6] (only helium for zero burnup).

$$a_{He} = 0.425 - 2.3 \times 10^{-4} \cdot T_{gas}$$

The heat transfer due to thermal radiation is calculated using the following expression used in FINIX code [5], in which two finite parallel grey surfaces is considered, where the radiation is leaving the first body and is directly intercepted or received at the second surface, as:

$$h_r = \frac{\sigma_{SB}}{\frac{1}{\epsilon_f} + \frac{R_{fo}}{R_{ci}}\left(\frac{1}{\epsilon_c} - 1\right)} \cdot \frac{\left({T_{fo}}^4 - {T_{ci}}^4\right)}{\left(T_{fo} - T_{ci}\right)} \tag{15}$$

where
ϵ_f = emissivity of the fuel outer surface
ϵ_c = emissivity of the cladding inner surface
σ_{SB} = Stefan-Boltzmann constant ($5.67 \times 10^{-8}\ \mathrm{W/m^2K^4}$)

The emissivity is taken as per FINIX code [5]:

$$\epsilon_f = 0.78557 + 1.5263 \times 10^{-5}\, T_f$$

$$\epsilon_c = 0.809$$

Where the temperature is in Kelvin, and the correlation for ϵ_f is the same as used in FRAPTRAN code and ϵ_c is taken from FEMAXI code [7] and implemented in FINIX code and preferred over FRAPTRAN correlation, which requires the information for cladding oxide thickness. In this proposed model we considered the cladding of Zircaloy with 1% Nb.

In zero burnup steady state case the gap is considered open, and in this case, the Eq. 11 is reduced to following with $h_c = 0$:

$$h_{gap} = h_g + h_r \tag{16}$$

Substituting h_g and h_r from Eq. 14 and Eq. 15 in the Eq. 16,

$$h_{gap} = \frac{k_{gas}}{\delta_{eff}} + \frac{\sigma_{SB}}{\frac{1}{\epsilon_f} + \frac{R_{fo}}{R_{ci}}\left(\frac{1}{\epsilon_c} - 1\right)} \cdot \frac{\left({T_{fo}}^4 - {T_{ci}}^4\right)}{\left(T_{fo} - T_{ci}\right)} \tag{17}$$

The value of h_{gap} from Eq. 17 is used in Eq. 12 for the calculation of temperature jump in fuel-cladding gap.

Table 1: Input parameter values

Region burnup $\left[\frac{MWd}{tU}\right]$	Linear pin power $[\frac{W}{cm}]$	Region average moderator temperature $[K]$	Fuel pellet radius $[cm]$	Gap thickness $[cm]$	Zircaloy thickness $[cm]$
0.0	191.72366	580.05	0.409575	0.008255	0.057150

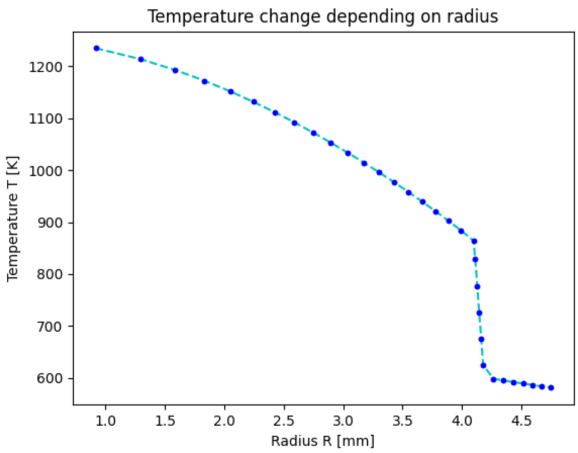

Figure 1: Temperature profile in the fuel rod as a function of distance from the centre of fuel rod for the initial values presented in Table 1.

5 CONCLUSION

In conclusion, this paper has presented preliminary work of an effective analytical model for heat transfer within a fuel rod, a critical component in nuclear reactors. By adopting one-dimensional radial geometry and assuming constant material properties along concentric rings, the model provides a practical tool for estimating temperature distributions in the fuel pellets, cladding, and the fuel-cladding gap. The study has elucidated complex heat transfer mechanisms, including gas conductance and radiative heat transfer, considering open gap condition. Through rigorous mathematical derivations, solutions for temperature profiles in the fuel and cladding are offered, highlighting their dependence on essential parameters.

As a future research direction, it is intended to extend this optimized model to investigate heat transfer in case of burnup and unsteady state for fuel pellet, cladding and the fuel-cladding gap (including closed-gap conditions), and in addition study of the axial dependence. This avenue holds significant potential for advancing our understanding of heat transfer in fuel rod. Preliminary integral effects validation results indicated that the optimized model in case of closed gap results in a reduction of fuel temperature due to enhanced heat transfer across the gap [9].

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