

On the Choice of Corresponding Pressures in a Novel Fluid-to-Fluid Similarity Theory for Heat Transfer at Supercritical Pressure

Andrea Pucciarelli, Sara Kassem, Walter Ambrosini

Università di Pisa, Dipartimento di Ingegneria Civile e Industriale
Largo Lucio Lazzarino, 2
56122 Pisa, Italy

andrea.pucciarelli@dici.unipi.it , sara.kassem@phd.unipi.it , walter.ambrosini@unipi.it

ABSTRACT

A novel fluid-to-fluid similarity theory, developed in the past years and recently refined and better assessed, allows for achieving an appropriate scaling of heat transfer phenomena at supercritical pressures with different fluid systems. The theory is a sound contribution to define dimensionless groups constituting the boundary conditions to be imposed in designing similar heat transfer experiments for different fluids at supercritical pressure. This achievement is of great importance for broadening the basis of available data in view of the design of the supercritical water-cooled reactors (SCWRs).

Previous publications and forthcoming papers describe different details of the similarity theory, showing its success in front of CFD analyses made by DNS, LES and RANS calculations. The aspect dealt with in the present paper relates to the choice of the most appropriate pressure for the different fluids, a necessary step to be performed, conditioning the accuracy of the obtained similarity in the selected operating conditions. Recipes were suggested in previous works for achieving this result, but a systematic analysis of their consequences and their relation to the classical choices dictated by the corresponding state theory was not yet performed in detail. This analysis is proposed in the present paper, better clarifying the issue in view of future refinements of the rationale at the basis of the similarity theory.

1 INTRODUCTION

Designing water cooled reactors operated at supercritical pressure (SCWRs), as the single Generation IV concept based on the widespread light water reactor technology, still needs considerable additional information in terms of fluid heat transfer, hydraulic resistance and flow stability phenomena in terms of experimental data. In addition to specific national and European projects that resulted in basic textbooks highlighting different aspects of reactor design (see e.g., the well-known textbooks by Prof. Oka and collaborators [1], the one by Pioro and Duffey [2] and the one by Schulenberg and Starflinger [3]), two coordinated research projects led by the International Atomic Energy Agency (IAEA) provided plenty of additional information on current SCWR concepts and the status of the researches in the field [4-6].

While engineers are exploring suitable practical solutions to the problems faced in this field, mainly originating by the considerable changes in fluid properties across the pseudo-critical threshold (where fluids at supercritical pressure exhibit a sharp peak in specific heat), knowledge and predictive gaps do exist asking for further original research. One of the gaps clearly remained even in recent work is related to the prediction of heat transfer deterioration phenomena identified long ago [7] that cannot be captured conveniently by any of the existing

heat transfer correlations. Several reviews have been published about existing engineering correlations (see e.g., [2, 8-9]) which showed that the phenomenological and quantitative representations of deteriorated heat transfer are still poor in front of the richness of behaviours observed in experimental data. The situation was sometimes considered rather desperate in terms of establishing a rationale for defining correlations on the basis of sound physical considerations [10]; notwithstanding the excellent work that has been done in the past to understand the physical features of deteriorated heat transfer [7, 11], still accurate quantitative consequences in terms of correlations have not yet been achieved. Though some of the recently proposed correlations provide apparently reasonable fits of experimental data (see e.g., [12]), they really seem not to solve the problem of finding the right form following the observed trends of the Nusselt number along a channel with deteriorated heat transfer. Moreover, correlations including wall temperature dependent dimensionless groups (e.g., wall to bulk density ratios and similar) are also identified as critical factors because they may lead to difficulties in convergence as well as to the possible presence of multiple solutions [13]. These aspects are presently the subject of a specific work by the authors of this paper, whose first results are being published elsewhere [14].

The present paper is focused on a specific aspect of a recently proposed fluid-to-fluid similarity theory for heat transfer to supercritical pressure fluids. Basing on old ideas [15], the rationale evolved in steps defining criteria for establishing similarity among different supercritical fluid conditions, reaching finally a sufficient maturity mainly supported by DNS, LES and RANS computations [16-19]. Since many of the results were obtained in subsequent phases, a recent paper was aimed to propose an overall view of the underlying rationale, providing guidance for its application [20]. The reason why this similarity theory was considered successful enough with respect to previous ones (see, e.g., the review by Mouslim and Tavoularis [10]) is that CFD analyses performed with variously scaled tools were able to reproduce quantitatively the same trends in a specific originally proposed dimensionless form for different fluids; in addition, in dimensional form (i.e., in the realm of actual temperatures and heat transfer rates) the same heat transfer regimes were observed. This result is remarkable and of great usefulness for designing facilities making use of less demanding fluids in terms of values of critical pressures and temperatures with respect to water, which can anyway provide useful information for supercritical water reactor conditions. As an additional benefit, the vast amount of experimental data already collected up to now by working fluids other than water can be made useful for supercritical water reactor conditions, when appropriately scaled.

The specific aspect of the rationale considered herein is related to the choice of supercritical pressures for the different fluids in order to establish the similarity. The need to consider this aspect stems from the fact that the adopted rationale is based on the choice of corresponding pressures of the different fluids that achieve in a same dimensionless enthalpy window comparable values of the product of the Stanton number by the length over diameter ratio. This criterion is not necessarily matching with the classical specification of equal reduced pressures, $p_{red} = p/p_{crit}$. Since some degree of tolerance has been found in specifying the corresponding pressures, the work clarifies how different is the choice suggested by the present rationale with respect to the above referred classical criterion based on the corresponding state theory. After a summary presentation of the similarity theory, this issue is discussed hereafter, with examples from addressed cases referring to experimental data with water by Watts [21] and by Pis'menny et al. [22-23] and carbon dioxide data by Kline [24].

2 SUMMARY OF THE SIMILARITY RATIONALE AND OF ITS RESULTS

Figure 1 summarises in a sketchy way the main steps to be accomplished for finding the boundary and operating conditions to be imposed to a given simulant fluid in similarity with a

case concerning a given reference fluid. For further details, the reader is referred to previous publications mentioned in the references and in particular to the latest archival one [20], presenting updated features of the theory. In order to understand the information reported in

Figure 1, it is necessary to clarify that the main dimensionless variables adopted in the rationale concern the specific enthalpy and the density, defined in dimensionless form as $h^* = (h - h_{pc}) \beta_{pc} / C_{p,pc}$ and $\rho^* = \rho / \rho_{pc}$, where the subscript pc identifies pseudocritical conditions at a given pressure, i.e., the temperature at which the specific heat at constant pressure reaches a maximum. Since an almost unique relation for dimensionless density with respect to dimensionless specific enthalpy is found for several fluids of interest (see the plot c in Figure 1) and owing to the great importance of density differences in heat transfer regimes, similarity is defined as *a similar distribution of dimensionless density and specific enthalpy along and across a pipe*. To achieve this, imposing the same value of the inlet pseudo-subcooling and of the power-to-flow ratio in dimensionless form, namely $N_{SPC} = -h_{in}^*$ and $N_{TPC} = \dot{Q} \beta_{pc} / W C_{p,pc}$, represent necessary conditions. This is in fact needed to locate the operating conditions of both fluids in a same region in terms of fluid inlet density and of the subsequent expansion of the fluid along the channel, thus leading to corresponding bulk dimensionless density and specific enthalpy trends all along the pipe.

As it can be noted in Figure 1, the first step in the procedure is selecting the operating pressures for the different simulant fluids basing on the pressure of the reference one; this aspect is exactly the subject of discussion in this paper. Pictures a) and b) reported in the figure have the purpose to symbolically represent the rationale at the basis of the choice of pressure, referring to the identification of pressure levels that assure the most uniform distribution of the Prandtl number ratios (Pr/Pr_{pc}) as a function of the dimensionless enthalpy. Alternatively, guessing the Stanton number (St) on the basis of a rough estimate by a supposed applicable heat transfer correlation (a classical one for forced convection) may drive to the actual searched condition for establishing similar pressures. It must be noted that, though this criterion is quite critical and may look cumbersome to apply, our research group is using it ordinarily with just a little effort.

As known, the Prandtl number at the peak (occurring very close to the pseudocritical conditions) may change considerably as a function of the supercritical pressure, reaching very high values while approaching the critical pressure; this feature was utilised in different ways in past applications, i.e., selecting pressures that provided the same peak Pr for the different fluids or, as above suggested, trying to get comparable values of Pr/Pr_{pc} in the addressed region. The former rationale was adopted in a “local” approach [17] that resulted successful when operating conditions were in a window of dimensionless enthalpy close to the pseudocritical region, thus leading to fluids having very similar Prandtl numbers throughout the operating range. The latter rationale, instead, was applied when dealing with dimensionless enthalpy windows extending mostly in the liquid-like region, where the values of the Prandtl number of fluids are unavoidably different; this involves compensating this difference by changing the length over diameter ratios, obtaining a more “general” rationale for applying the theory [18].

Evidently, these ways of selecting pressure are different in principle from assuming an equal reduced pressure and the purpose of this paper is to establish a posteriori how much these two criteria can be considered deviating from each other. In STEP 3, the same Froude number and Reynolds number values are imposed at channel inlet; the former choice is dictated by the need to keep the same ratio of kinetic energy over gravity forces, while the second tries to obtain comparable turbulence levels in the two similar conditions: to achieve this equality, relations for determining the inlet velocity and diameter must be fulfilled, as reported in

Figure 1 for purpose of illustration of the STEP3.

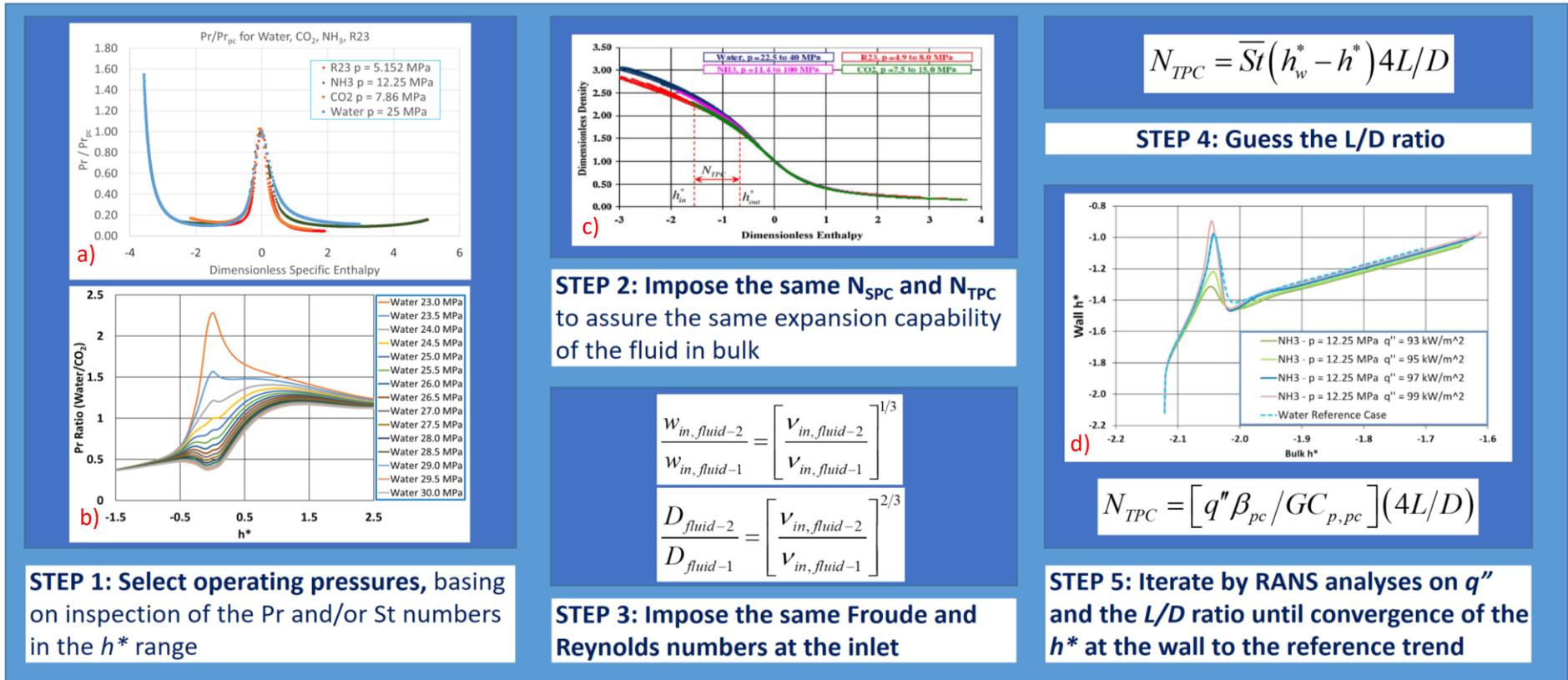


Figure 1: Description of the main steps in the similarity rationale

The formulation reported in Figure 1 for N_{TPC} above the short explanation of STEP 4 represents the dimensionless form of the Newton's law of convection; owing to the equality of the trends of the dimensionless enthalpies at the wall and in bulk in the two similar cases, any difference in the values of the average Stanton numbers in the two fluids must be compensated with a change of the pipe length over diameter ratio (L/D). Finally, STEP 5 involves an iterative procedure, in which the heat flux is progressively adjusted while changing also the L/D ratio, aiming to achieve a same trend of the dimensionless enthalpy at the wall as a function of the dimensionless enthalpy in bulk for the two conditions, as shown in Figure 1d. While referring again the reader to previous publications for sample applications and for better understanding the features of the similarity rationale [18-20], it is here recalled that the similarity allows for reproducing the same normal, enhanced, deteriorated and restored heat transfer conditions observed in an experiment with a given fluid also with other fluids. It therefore constitutes a unique and very effective theory for planning counterpart experiments, something lacking before its appearance, and up to now proven convincingly on the basis of CFD calculations at different levels of resolution. A striking example of this similarity is provided by the sequence of plots shown in Figure 2 for four different fluids and for some specific reference cases all obtained with CO₂ in the data by Kline [24]. Many more cases have been analysed obtaining quite convincing results, in analogy with those shown herein.

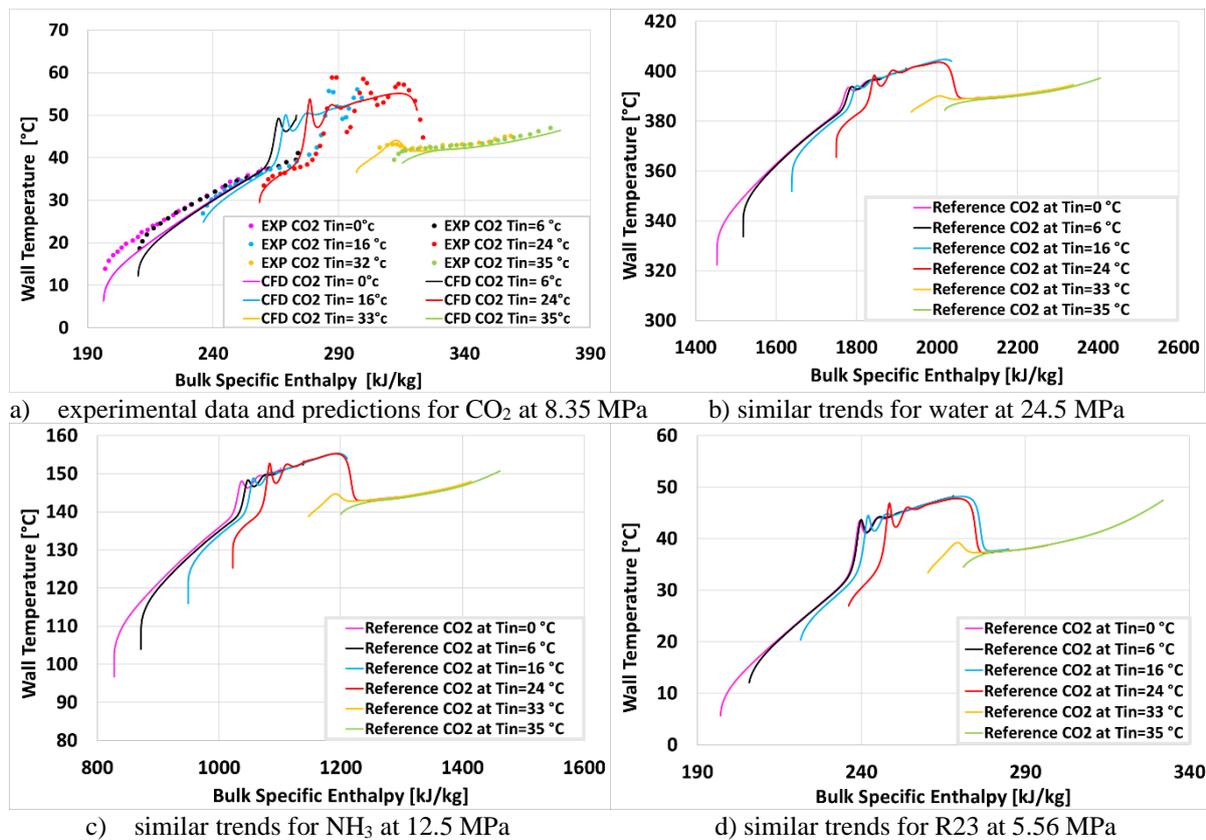
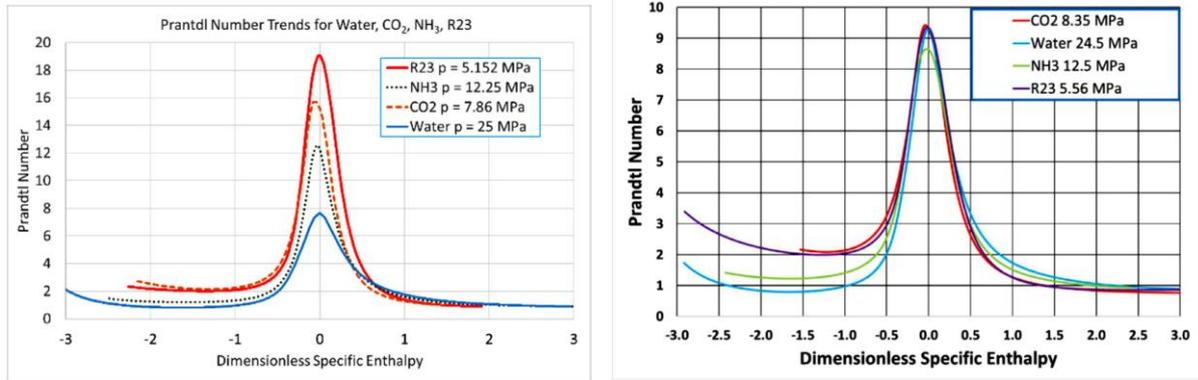


Figure 2: Similar trends obtained by RANS calculation for the reference CO₂ experiments by Kline [24] at 8.35 MPa, 4.6 mm ID, $q''=18$ kW/m², $G=300$ kg/m²s obtained adopting different fluids

3 REDUCED PRESSURES AND PRESSURES ADOPTED IN THE RATIONALE

The pressures selected for the above sketched applications and for the others which are not shown here were selected with the criterion of finding a sufficiently uniform ratio of the Prandtl numbers in the dimensionless enthalpy window addressed by the data. As previously mentioned, this led to different choices for cases in which most of the evolution of temperatures in bulk and at the wall were in the liquid-like region and for conditions that were mostly trans-pseudocritical, i.e., involving both the liquid-like and the gas-like regions.



a) at pressures selected for Watts' data

b) at pressures selected for Kline's data

Figure 3: Values of the Prandtl numbers at pressures selected for achieving the best results with different data

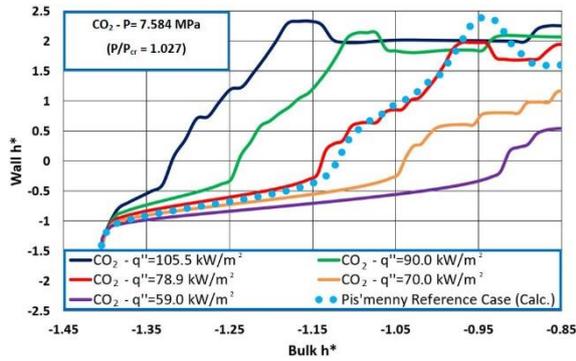
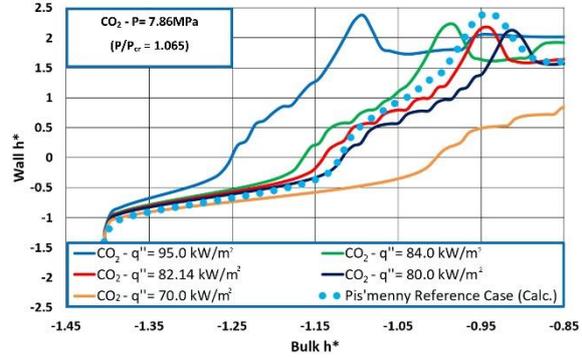
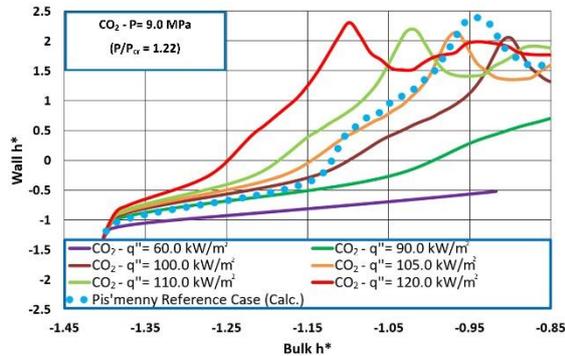
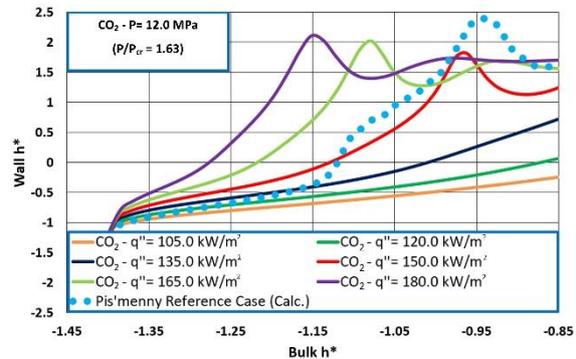
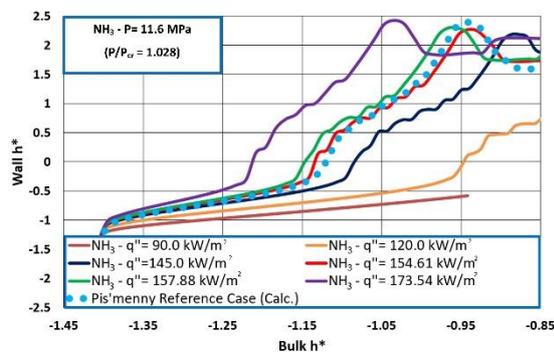
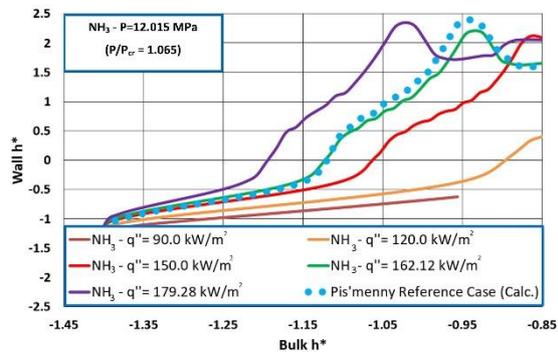
These choices are clearly illustrated in Figure 3. Watts' data are mostly characterised by an evolution of the bulk temperature and partly of the wall one in the liquid-like region, while Kline's data mostly span over the two regions, crossing the pseudocritical threshold. In the former case, the rationale followed was mostly to keep as far as possible a constant ratio of the molecular Prandtl number in the considered region, choosing pressures that provided a value of Pr at the peak having nearly the same ratio observed in the liquid like region: this is clearly shown in Figure 3a. Figure 3b, instead, shows that the rationale for Kline's data was mostly dictated by the attempt to find the same value of the molecular Prandtl number at the peak. It must be remarked that both strategies worked reasonably well in the respective domains, though some flexibility was noted in their application, allowing for slight deviations in pressures with little worsening of the degree of similarity predicted by CFD codes.

Table 1 summarises the values of the actually used pressures for the two cases and of the corresponding reduced pressures. As it can be noted, in the case of Watts' data the pressures selected for the fluids other than the reference one (water) have a reduced pressure mostly in a same range, but well below the one of the reference; on the other hand this difference is much less remarkable in the case of the choices made for Kline's data. This diversity of the two cases seems to be related to the mentioned different windows in the dimensionless enthalpy, being mostly on the negative side for Watts' data (liquid-like region) and more balanced between the positive and the negative side for Kline's data (across the pseudo-critical region). In both cases, anyway the rule to start the search for the best pressure for similarity by using the corresponding state theory (i.e., keeping an equal reduced pressure) seems to lead to possibly good guesses, though not necessarily to the best ones.

The plots in Figure 4 to Figure 11 refer instead to an experimental case by Pis'menny et al. [22-23] with water at 23.5 MPa ($p/p_{cr} = 1.065$), $q'' = 390 \text{ kW/m}^2$, $T_{in} = 300 \text{ }^\circ\text{C}$ and $G = 509 \text{ kg/m}^2\text{s}$.

Table 1. Adopted pressures for Watts' and Kline's data and corresponding reduced values

Watts' data	p_{red}	Kline's data	p_{red}
Water (Ref.): 25 MPa	1.133067	CO ₂ (Ref): 8.35 MPa	1.13185
CO ₂ : 7.86 MPa	1.06543	Water: 24.5 MPa	1.110406
NH ₃ : 12.25 MPa	1.07806	NH ₃ : 12.5 MPa	1.100062
R23: 5.152 MPa	1.066225	R23: 5.56 MPa	1.150662

Figure 4: Dimensionless enthalpy trends at different values of the heat flux for CO₂ at 7.584 MPaFigure 5: Dimensionless enthalpy trends at different values of the heat flux for CO₂ at 7.86 MPaFigure 6: Dimensionless enthalpy trends at different values of the heat flux for CO₂ at 9.0 MPaFigure 7: Dimensionless enthalpy trends at different values of the heat flux for CO₂ at 12.0 MPaFigure 8: Dimensionless enthalpy trends at different values of the heat flux for NH₃ at 11.6 MPaFigure 9: Dimensionless enthalpy trends at different values of the heat flux for NH₃ at 12.015 MPa

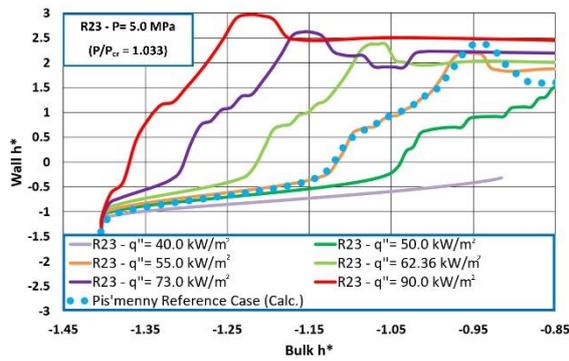


Figure 10: Dimensionless enthalpy trends at different values of the heat flux for R23 at 5.0 MPa

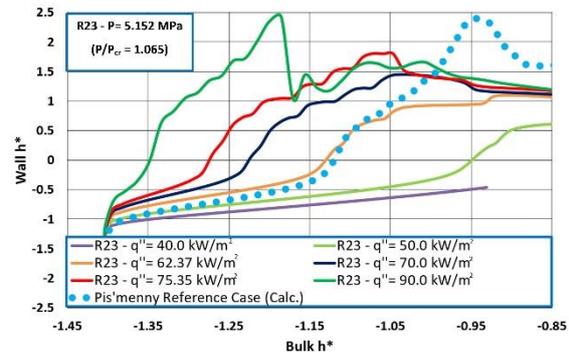


Figure 11: Dimensionless enthalpy trends at different values of the heat flux for R23 at 5.152 MPa

Since the reduced pressure for the water reference case is 1.065, for all the fluids the results obtained with the same reduced pressure of the reference case are reported in the plots together with the pressure which was found to be the best one for scaling. For CO_2 (Figure 4 to Figure 7) further attempts were made, showing that, at least for these boundary conditions, the applicability range may be large enough to allow for some degree of flexibility in selecting pressure. This partially holds also for NH_3 (Figure 8 and Figure 9), where an additional lower value of pressure seemed not to worsen the results in terms of similarity. For R23, instead, it seems that the applicability range is much smaller: at the same reduced pressure of the base case the reference trend could not be obtained and the operating pressure was then decreased.

4 CONCLUSIONS

This paper highlighted a specific detail of a theory developed for establishing fluid-to-fluid similarity in the complex phenomenon of heat transfer to fluids at supercritical pressures. The work was suggested by the fact that the rationale for finding corresponding pressures for the different fluids is based on the trends of the molecular Prandtl number and/or of a guess of the Stanton number, evaluated basing on some hypothesized heat transfer correlation. Therefore, since these criteria are not necessarily related to the usual assumption of matching reduced pressures, a check of the relation of the two independent rationales was in order.

The obtained results show that the choice of pressure is a problem-dependent one and that the selection of equal reduced pressures may be a rough guidance, but it does not necessarily solve the problem of establishing good correspondence between the obtained trends in all the conditions. A rather reassuring aspect is that some degree of freedom in choosing pressures, in reasonably small ranges, makes the problem relatively less stiff than it could be assumed.

It is anyway clear that more cases should be addressed to draw final conclusions and that experimental data comparisons, though absolutely difficult to be performed without dedicated experimental campaigns, could say something more in regard to the different details of a rationale that has received anyway a reassuring support by numerical predictions. A further interesting problem to be investigated is the relation of the inlet temperatures defined by the criterion of the equality of the N_{SPC} number with the one based on the use of reduced temperatures.

In summary, though a considerable work has been already done in relation to the present similarity theory since its proposal, still interesting aspects deserve to be investigated to acquire better awareness of its characteristics and potential. This paper contributed to clarify one of the issues in the proposed rationale, providing greater insight into its characteristics.

5 REFERENCES

- [1] Y. Oka, S. Koshizuka, Y. Ishiwatari, and A. Yamaji, *Super Light Water Reactors and Super Fast Reactors*, 1st ed. Springer, Boston, MA, 2010.
- [2] I. L. Pioro and R. B. Duffey, *Heat Transfer and Hydraulic Resistance at Supercritical Pressures in Power–Engineering Applications*. New York: ASME Press, 2007.
- [3] Schulenberg, T.; Starflinger, J. *High Performance Light Water Reactor: Design and Analyses*; KIT Scientific Publishing: Karlsruhe, Germany, 2012.
- [4] International Atomic Energy Agency. *Heat Transfer Behaviour and Thermohydraulics Code Testing for Supercritical Water Cooled Reactors (SCWRs)*; IAEA-TECDOC-1746; IAEA: Vienna, Austria, 2014.
- [5] International Atomic Energy Agency. *Status of Research and Technology Development for Supercritical Water-Cooled Reactors*; IAEA-TECDOC-1869; IAEA: Vienna, Austria, 2019.
- [6] International Atomic Energy Agency. *Understanding and Prediction of Thermohydraulic Phenomena Relevant to Supercritical Water Cooled Reactors (SCWRs)*; IAEA-TECDOC-1900; IAEA: Vienna, Austria, 2020.
- [7] Jackson, J.; Hall, W. Influences of buoyancy on heat transfer to fluids flowing in vertical tubes under turbulent conditions. *Turbul. Forced Convect. Channels Bundles* 1979, 2, 613–640.
- [8] Cheng, X.; Schulenberg, T. *Heat Transfer at Supercritical Pressures–Literature Review and Application to an HPLWR*; FZKA: Karlsruhe, Germany, 2001; Volume 6609.
- [9] Huixiong Li, Xiangfei Kong, Xianliang Lei, Qian Zhang, Changjiang Liao, Lingtong Gou and Dongfeng Sun (October 19th 2019). *Heat Transfer Correlations for Supercritical Water in Vertically Upward Tubes*, *Advanced Supercritical Fluids Technologies*, Igor Pioro, IntechOpen, DOI: 10.5772/intechopen.89580. Available from: <https://www.intechopen.com/chapters/69637>
- [10] Mouslim, A.; Tavoularis, S. Direct tests of fluid-to-fluid scaling expressions for supercritical heat transfer in tubes. *Int. J. Heat Mass Transf.* 2020, 147, 118862.
- [11] Licht, J.; Anderson, M.; Corradini, M. Heat transfer to water at supercritical pressures in a circular and square annular flow geometry. *Int. J. Heat Fluid Flow* 2008, 29, 156–166.
- [12] S. J. Mokry, I. L. Pioro, A. Farah, and K. King, “ICONE19-43876 Updated Heat Transfer Correlations for Supercritical Water-Cooled Reactor Applications,” *Proc. Int. Conf. Nucl. Eng.*, vol. 2011.19, no. 0, p. ICONE1943–ICONE1943, 2011.
- [13] H. Zahlan, L. Leung, Y. Huang and G. Liu, Convection heat transfer correlations for multiple geometries and fluids at supercritical pressure, *CNL Nuclear Review*, Vol 7, Number 1, June 2018.
- [14] Kassem, S., Pucciarelli, A. and Ambrosini W., Considerations about Correlations for Heat Transfer to Supercritical Pressure Fluids Spanning All Regimes, Submitted at The 19th International Topical Meeting on Nuclear Reactor Thermal Hydraulics (NURETH-19) Log nr. 19001, Brussels, Belgium, March 6 - 11, 2022.
- [15] Ambrosini, W., Discussion of similarity principles for fluid-to-fluid scaling of heat transfer behaviour at supercritical pressures. *Nucl. Eng. Des.* 241 (2011), 5149–5173.
- [16] Pucciarelli, A., Ambrosini, W., Fluid-to-fluid scaling of heat transfer phenomena with supercritical pressure fluids: Results from RANS analyses, *Annals of Nuclear Energy* 92 (2016) 21–35.
- [17] Pucciarelli, A., He, S., Ambrosini, W., A successful local fluid-to-fluid similarity theory for heat transfer to supercritical pressure fluids: merits and limitations, *International Journal of Heat and Mass Transfer*, Volume 157, August 2020, 119754
- [18] Pucciarelli, A., Ambrosini, W., A successful general fluid-to-fluid similarity theory for heat transfer at supercritical pressure, *International Journal of Heat and Mass Transfer*, Volume 159, October 2020, 120152
- [19] Kassem, S.; Pucciarelli, A.; Ambrosini, W. Insight into a fluid-to-fluid similarity theory for heat transfer at supercritical pressure: Results and perspectives. *Int. J. Heat Mass Transf.* 2021, 168, 120813.
- [20] Pucciarelli, A., Kassem, S., Ambrosini, W. Overview of a Theory for Planning Similar Experiments with Different Fluids at Supercritical Pressure. *Energies* 2021, 14, 3695. <https://doi.org/10.3390/en14123695>

- [21] Watts, M.J. Heat Transfer to Supercritical Pressure Water–Mixed Convection with Upflow and Downflow in a Vertical Tube. Ph.D. Thesis, University of Manchester, Manchester, UK, 1980.
- [22] Pis'menny, E.N., Heat transfer to supercritical water in gaseous state or affected by mixed convection in vertical tubes. In Proceedings of ICONE14 "International Conference of Nuclear Engineering". 2005, Miami, Florida, USA.
- [23] Pis'menny, E.N., Razumovskiy, V.G., Maevskiy, E.M.,. Experimental study on temperature regimes to supercritical water flowing in vertical tubes at low mass fluxes, Proceedings of the International Conference GLOBAL-2005 Nuclear Energy Systems for Future Generation and Global Sustainability, Tsukuba, Japan, October 9-13, 2005, paper no. 519.
- [24] Kline, N. An Experimental Study on Heat Transfer Deterioration at Supercritical Pressures. Master's Thesis, Ottawa-Carleton Institute for Mechanical and Aerospace Engineering, University of Ottawa, Ottawa, ON, Canada, 2017.

ACKNOWLEDGMENTS

The latest applications of the theory reported in this paper were performed in the frame of the EU ECC-SMART project: this project received funding from the Euratom research and training programme 2019–2020 under grant agreement no. 945234. The similarity theory was developed over the timespan of more than a decade, the latest developments occurring in the first months of 2020 during the first wave of the COVID-19 pandemic. The IAEA is gratefully acknowledged for the involvement granted in two subsequent CRPs on the SCWR thermal hydraulics in the past years.