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Fluid-Structure Interaction, Congested Media, And Wavelets : A Multi-Scale And Homogenized Approach

Samy Mokhtari[1,2], **Guillaume Ricciardi**[1], **Vincent Faucher**[1]
[1]CEA, DES, IRESNE, DTN, Cadarache
F-13108 Saint-Paul-Lez-Durance, France
[2]MAST-EMGCU, Université Gustave Eiffel, IFSTTAR
F-77454 Marne-la-Vallée, France
samy.mokhtari@univ-eiffel.fr, guillaume.ricciardi@cea.fr, vincent.faucher@cea.fr

Pierre Argoul, Lucas Adelaïde
MAST-EMGCU, Université Gustave Eiffel, IFSTTAR
F-77454 Marne-la-Vallée, France
pierre.argoul@univ-eiffel.fr, lucas.adelaide@univ-eiffel.fr

ABSTRACT

Computing a flow within a highly congested solid medium is still nowadays an important scientific issue in many research fields, such as nuclear engineering. Indeed, confronted with an overwhelming number of interfaces, the classical Fluid-Structure Interaction approach would inevitably lead to cumbersome computations. This important issue of interfaces can also be coupled with multi-scale phenomena, caused both by the fluid and the solid medium geometry. In order to deal with these interfaces and multi-scale problematics, this work presents a multi-scale and homogenized model able to account for an inviscid compressible flow within a congested solid medium. An original use of Continuous Wavelet Transform here allows to derive spatially-filtered PDEs governing an equivalent/homogenized fluid. The numerical computation of the homogenized fluid PDEs allows to reconstruct (thanks to an inverse wavelet transform), at each time step, the pressure field in the real fluid, which leads to the dynamic load applied to the solid medium. This important step, validated on 2D reference numerical solutions with steady solid obstacles, thus opens the way to a coupled fluid-structure solver.

1 INTRODUCTION

The current work finds its starting point in the study of the mechanical consequences of accidental scenarios for Pressurized Water Reactors (PWR), with a focus on the propagation of transverse pressure waves through the fuel assemblies of a nuclear core (see Figures 1a-2). Such a phenomenon, called Loss Of Cooling Accident (LOCA), originates from a failure in one of the pipes of the pressurized primary loop (155 bar). The Physics of interest thus requires to compute a compressible flow within a highly congested solid medium, here the fuel assemblies. Confronted with an overwhelming number of interfaces, the classical Fluid-Structure Interaction (FSI) approach [1, 2, 3], which relies on an explicit representation of all the interfaces, would be too cumbersome. This important issue of interfaces is also here coupled with multi-scale phenomena: a wide range of spatial scales is for instance contained within a viscous turbulent flow, possibly entangled with the different spatial scales of the congested solid medium. To tackle both the interface and multi-scale problematics, one may try to simplify the description of the problem at hand, by coarsening the scale of representation. A porous media approach has for instance already been proposed in [4, 5, 6] for the study of a PWR core dynamics in response to a seismic transient.

(a) (b)

Figure 1: Fuel assemblies design: overview (1a) and spacer grid (1b).

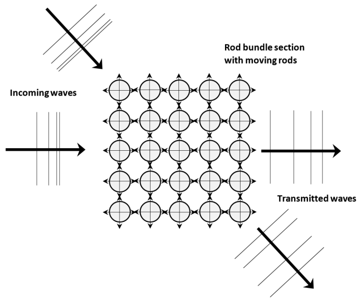


Figure 2: Sketch of a 2D pressure wave propagating through a fuel assembly cross section.

This quest for an affordable and ”optimal” scale of representation is a major trend that crosses multiple scientific and engineering communities : signal and image processing (data compression), continuum media modeling (e.g. heterogeneous materials, porous media, turbulence, two-phase flows), problems defined in high dimensional spaces (e.g. kinetic theory, molecular dynamics. . .). Regarding continuum media problems, one could distinguish two major trends in literature : on the one hand, approaches willing to build brand new equations describing an equivalent and simplified medium (e.g. analytical homogenization of heterogeneous materials, Large Eddy Simulation, Variational MultiScale Method...), and on the other hand, approaches which leave the original PDEs intact, and rather focus on speeding up the computations (computational homogenization, adaptive numerical methods, Reduced Order Modeling...). To the authors’ knowledge, all these different methods are still currently facing either theoretical or numerical challenges : treatment of boundary conditions, closure equations, strict scale separation, periodicity, existence of a Representative Volume Element, nonlinearities, insights on the fine-scale solution, separated representation of the unknowns. . .

Considering this extensive state of the art, the present work puts forward an analytical multi-scale and homogenized model able to account for the propagation of an inviscid compressible flow within a congested solid medium. In order to build a self-sustained model, bypassing the multiple limitations previously highlighted, this work promotes an original use of Continuous Wavelet Transform (CWT). By applying, by means of a convolution product, a well-designed wavelet (or scaling function) to the fluid Partial Differential Equations (PDEs), the model results in spatially-filtered PDEs governing an equivalent/homogenized fluid in the whole {fluid + solid} domain. Furthermore, thanks to an inverse wavelet transform, the model is able to connect analytically resolved (i.e. the homogenized fluid) and unresolved (i.e. the real fluid) scales. This wavelet-based closure equation allows on the one hand, to rigorously transfer the original fluid boundary conditions into the homogenized fluid, and on the other hand to explicitly handle nonlinearities. The numerical computation of the homogenized fluid PDEs then allows to reconstruct, at each time step, the pressure field in the original fluid, which leads to the dynamic load applied to the solid medium.

In this work, the choice has been made to focus the homogenization process on the fluid, as it occupies a connected domain in the geometry of interest. Furthermore, CWT is hereafter applied in a 2D formalism. This work indeed focuses on the propagation of transverse pressure waves through the cross section of fuel assemblies, as displayed in Figure 2.

The following of this paper is organized as follows: section 2 presents the key ingredients of the wavelet-based homogenization process. Section 3 then briefly illustrates and discusses the model capabilities with numerical experiments involving 2D shock waves propagating through solid obstacles. The final section is then dedicated to a conclusion.

2 MODELING

2.1 Modeling at the microscopic scale

To begin this section, let us state the modeling at the microscopic scale. As an illustration for the problem at study, let us consider the 2D geometry displayed on the following Figure 3:

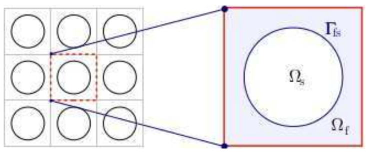


Figure 3: Illustration of a 2D {fluid + solid} geometry.

The whole {fluid + solid} domain thus contains a fluid sub-domain Ω_f, which is an open bounded and connected space of $\mathbb{R}^2$, and a solid sub-domain Ω_s, which is an open bounded and disconnected space of $\mathbb{R}^2$. The boundaries $\partial\Omega_f$ and $\partial\Omega_s$ are generally assumed smooth (typically C^1) in order to properly define the outward normal vectors. It is important to note that no periodicity or scale separation assumption on the solid domain Ω_s (or on Ω_f) are needed in the design of the model. The problem geometry being stated, let us now focus on the fine-scale modeling of the solid and fluid media :

- as we are here mainly interested in the homogenization process of the fluid, and as spacer grids tend to maintain a constant distance between neighboring disks, the global 2D array will be simplified and considered as a rigid body animated with two degrees of freedom, governed by the following differential equation:

$$\forall i \in \{1, 2\}, \ddot{U}_i + 2\xi\omega_0\dot{U}_i + \omega_0^2 U_i = \frac{1}{m} \times \left(\underline{F}_{F\to S} \cdot \underline{e_i}\right), \tag{1}$$

where $\left(\underline{e_1}, \underline{e_2}\right)$ is the orthonormal Cartesian basis of $\mathbb{R}^2$, $\underline{U} = \begin{pmatrix} U_1 & U_2 \end{pmatrix}^T$ is the displacement (m), m is the mass (kg), ω_0 is the system eigenfrequency (rad.s^{-1}), ξ is the (dimensionless) damping coefficient and $\underline{F}_{F\to S}$ is the force (N) applied by the fluid to the whole array of disks.

- the water flow will be considered as a monophasic compressible and inviscid fluid, satisfying Euler compressible equations and a barotropic equation of state:

$$\left\{\begin{array}{ccll} \partial_t\rho + div\left(\rho\underline{v}\right) & = & 0 & \text{in } \Omega_f(t), \\ \partial_t\left(\rho\underline{v}\right) + \underline{div}\left(\rho\underline{v}\otimes\underline{v}\right) & = & -\underline{\nabla}\, p & \text{in } \Omega_f(t), \\ \partial_t\left(\rho e\right) + div\left(\left(\rho e + p\right)\underline{v}\right) & = & 0 & \text{in } \Omega_f(t), \end{array}\right. \tag{2}$$

$$p = p_{ref} + c_{son}^2\left(\rho - \rho_{ref}\right), \tag{3}$$

where ρ is the fluid density $(\mathrm{kg.m}^{-3})$, $\underline{v}$ the velocity field $(\mathrm{m.s}^{-1})$, p the pressure field (Pa), e the specific total energy $(\mathrm{J.kg}^{-1})$, ρ_{ref} a reference density, $p_{ref} = p\left(\rho_{ref}\right)$ the corresponding reference pressure, and $c_{son} = \sqrt{\partial_\rho p}$ the sound velocity in the fluid. It shall here be noted that, when studying a barotropic fluid, it is not necessary to solve the energy balance equation. Thus, only the mass and momentum balance equations will be hereafter considered.

The above equations are completed with classical no-penetration kinematic boundary conditions, and a dynamic boundary condition for the pressure on the interfaces.

2.2 Wavelet-based homogenization

The fine-scale modeling being stated, the wavelet-based homogenization process now relies on the application of Continuous Wavelet Transform (CWT) to the fluid conservation laws and equation of state. This is done by writing the convolution products between a wavelet family $\{\Psi_s\}_{s>0} = \left\{\frac{1}{s}\Psi\left(\frac{\cdot}{s}\right)\right\}_{s>0}$ (where s is the scale/dilation parameter of the wavelet) and the fluid equations, as formally illustrated by equations (4-5) below:

$$\left(\widetilde{\Psi}_s^*\right) * \left\{\begin{array}{ccc} \partial_t\rho + div\left(\rho\underline{v}\right) & = & 0, \\ \partial_t\left(\rho\underline{v}\right) + \underline{div}\left(\rho\underline{v}\otimes\underline{v}\right) & = & -\underline{\nabla}\, p, \end{array}\right. \tag{4}$$

$$\widetilde{\Psi}_s^* * \left\{p = p_{ref} + c_{son}^2\left(\rho - \rho_{ref}\right)\right\}. \tag{5}$$

As wavelets (respectively scaling functions) act as band-pass (resp. low-pass) filters in the spectral domain, such a convolution product will result in spatially-filtered PDEs describing an equilvalent/homogenized fluid. For an introduction to Continuous Wavelet Transform (CWT), its properties and applications, the interested reader may refer to the works [7, 8, 9].

To apply such a convolution product on PDEs initially defined on a bounded domain Ω_f, and possibly exhibiting non-smooth solutions (e.g. shock waves), the original PDEs first have to be extended in a weak sense to $\mathbb{R}^2$. This extension mostly relies on distribution theory and Green's formula for integration by parts. Then, one can write, in a weak sense, the convolution product between the wavelets and the extended fluid PDEs. All these steps are thoroughly described in [10], and will be reinforced for a generic continuum medium problem in an upcoming article. Assuming that the previous procedure is followed, one may then obtain, with a real and isotropic wavelet Ψ (or its associated scaling function), the following spatially-filtered PDEs:

Consider $T > 0$. $\forall \underline{u} \in \Omega_f \cup \Omega_s, \forall t \in [0, T[, s > 0$:

$$\partial_t \mathcal{W}[\rho](s, \underline{u}, t) + div\left(\mathcal{W}\left[\rho \underline{v}\right]\right)(s, \underline{u}, t) = -\int_{\partial \Omega_s} \widetilde{\Psi}_s^*(\underline{u} - \underline{\sigma})[\rho]_S^F \left[\partial_t \underline{U} \cdot \underline{n}_{F \to S}\right](\underline{\sigma}, t)\, \mathrm{d}\underline{\sigma}. \quad (6)$$

$$\begin{aligned} &\partial_t \mathcal{W}\left[\rho \underline{v}\right](s, \underline{u}, t) + \underline{div}\left(\mathcal{W}\left[\rho \underline{v} \otimes \underline{v}\right]\right)(s, \underline{u}, t) + \underline{\nabla} \mathcal{W}[p](s, \underline{u}, t) \\ &= -\int_{\partial \Omega_f} \widetilde{\Psi}_s^*(\underline{u} - \underline{\sigma})\, [p]_{^cF}^F (\underline{\sigma}, t) \cdot \underline{n}_{F \to {^cF}}(\underline{\sigma}, t)\, \mathrm{d}\underline{\sigma} \\ &\quad - \int_{\partial \Omega_s} \widetilde{\Psi}_s^*(\underline{u} - \underline{\sigma})\, [\rho \underline{v}]_S^F (\underline{\sigma}) \left[\partial_t \underline{U} \cdot \underline{n}_{F \to S}\right](\underline{\sigma}, t)\, \mathrm{d}\underline{\sigma}. \end{aligned} \quad (7)$$

$$\mathcal{W}[p](s, \underline{u}, t) = c_{son}^2 \mathcal{W}[\rho](s, \underline{u}, t). \quad (8)$$

where $\mathcal{W}[f](s, \cdot) = \left(f * \widetilde{\Psi}_s^*\right)$ denotes the wavelet coefficient of f for a given dilation/scale parameter s, and $[f]_2^1 = f_1 - f_2$ denotes the jump of discontinuity of f across a boundary.

In these PDEs, it is important to emphasize the role played by the function

$$\widetilde{\underline{F}}_{S \longrightarrow F}(s, \underline{u}, t) := -\int_{\partial \Omega_f} \widetilde{\Psi}_s^*(\underline{u} - \underline{\sigma})\, [p]_{^cF}^F (\underline{\sigma}, t) \cdot \underline{n}_{F \to {^cF}}(\underline{\sigma}, t)\, \mathrm{d}\underline{\sigma}, \quad (9)$$

which is a body force (per unit of length) applied by the underlying solid obstacles (and the outer boundaries) to the homogenized fluid, across the whole space $\mathbb{R}^2$. It represents the resistance that encounters the original fluid when flowing through the solid medium and impacting the outer boundaries. The homogenization process thus transforms contact forces, localized on the fluid-structure interfaces and outer boundaries, into a body force.

Besides, one can notice that the body force (9) depends on the original pressure field p, which contains all the possible spatial scales that could be caught with a DNS computation of the original fluid PDEs (a similar remark stands for the other boundary integrals). A closure expression between the unresolved and resolved scales of the pressure field is thus required, as in any homogenization or multi-scale method. Fortunately, conversely to plain filtering or averaging techniques, CWT offers an inverse transform which brings us an analytical closure expression, that one could formally write for simplicity :

$$p = CWT^{-1}\left[\left(\mathcal{W}[p](s, \cdot)\right)_{s>0}\right]. \quad (10)$$

By limiting the number of wavelet coefficients retained in equation (10), it is thus possible to reconstruct (up to an approximation), at each time step, the microscopic pressure field on the fluid inner (and outer if necessary) boundaries, and to evaluate the body force applied by the underlying solid obstacles. The same equation (10) also allows to reconstruct the density ρ and velocity $\underline{v}$, thus leading to the nonlinear convective term $\rho \underline{v} \otimes \underline{v}$ and the corresponding wavelet coefficients $\mathcal{W}\left[\rho \underline{v} \otimes \underline{v}\right](s, \cdot)$. CWT thus allows to tackle nonlinearities !

3 RESULTS AND DISCUSSION

The key ingredients of the model being recalled, let us now have a look at some preliminary numerical results. The homogenized fluid PDEs are hereafter discretized in space via a 1st order finite-volume method with directional splitting. Thanks to the homogenization process, this finite-volume method can be associated to a plain 2D regular Cartesian grid. Considering the fast-transient phenomenon at study, an Euler explicit scheme is chosen for the time discretization. The numerical fluxes at the interfaces are computed using an approximate Riemann solver, namely Rusanov flux.

In order to confront the wavelet-based model with the Physics of interest, reference shock waves propagating through a 10×10 steady array of disks are computed at the DNS scale with EUROPLEXUS software, a fast-transient dynamics code for fluids and structures. Figures 4-5 hereafter confront (with a 10 VS 1 bar initial pressure discontinuity) the reference pressure field with the reconstructed pressure field, obtained by solving the spatially-filtered PDEs (6-7-8) with the Mexican hat scaling function (instead of its wavelet), i.e. a low-pass filter of cutoff scale s_0. The constant C_{stab} below refers to the safety margin on the time step with respect to the Courant-Friedrichs-Lewy stability condition. The constant h (= disk radius/4) refers to the mesh size. With a steady array of disks, the wavelet-based model is thus able to reconstruct a horizontal pressure profile which closely fits the reference data. Nevertheless, a high frequency noise can be witnessed within the array of disks (delimited by the vertical black lines in Figure 5 below), which is explained by an aliasing phenomenon induced by the scaling function. This noise can be reduced by increasing the cutoff scale s_0.

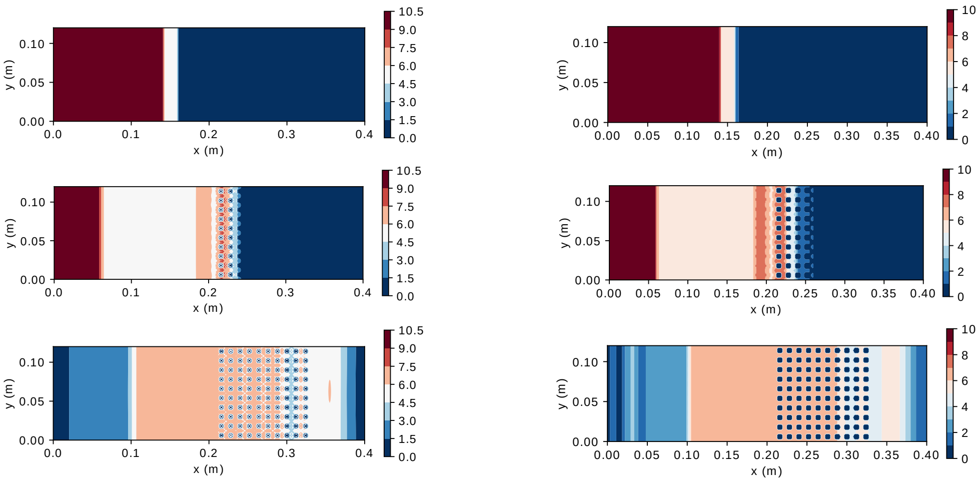


Figure 4: Reconstructed (left) VS reference (right) pressure fields snapshots - $s_0 = 0.415h$

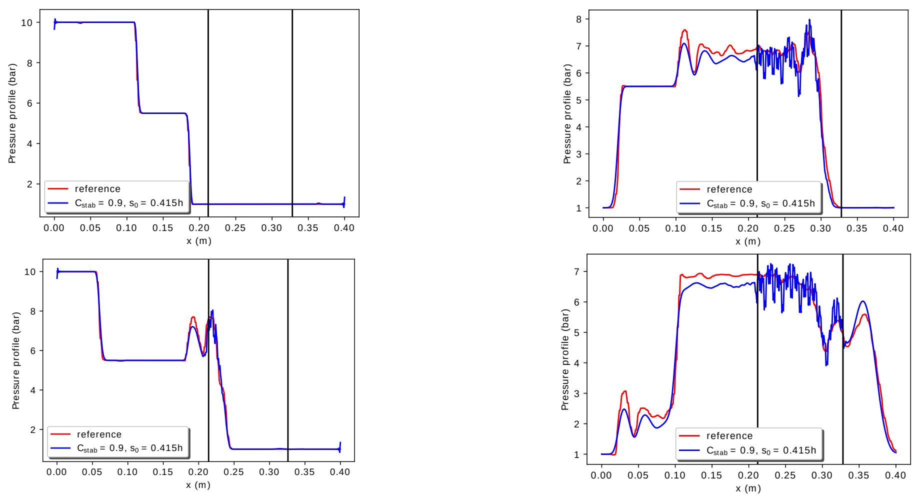
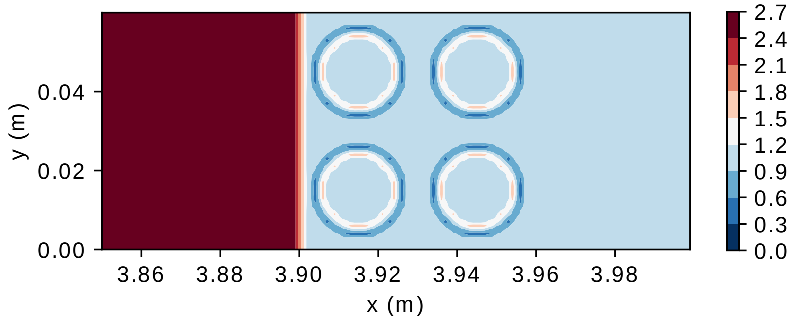
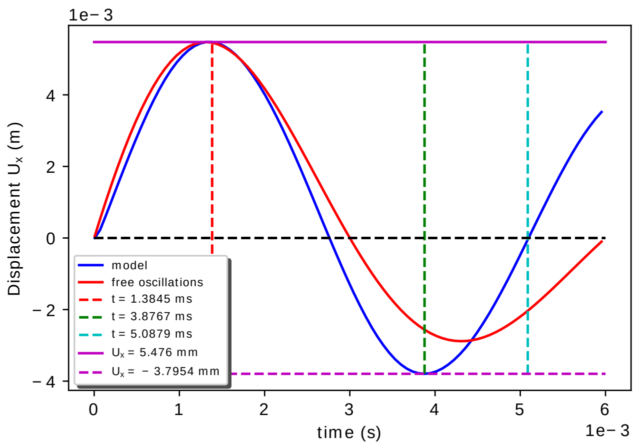


Figure 5: Horizontal pressure profile - 10×10 array - $C_{stab} = 0.9$ - $s_0 = 0.415h$

Preliminary numerical tests have also been realised to assess the model capability to treat the coupling with moving solid obstacles. In Figure 6 below, the propagation of a 2D shock wave through a 2×2 moving array of disks (radius = 10 mm) is considered. The aim of this test was simply to recover, with the wavelet-based model equations, the theoretical behavior of a linear oscillator that would be submitted to the (reconstructed) force applied by the fluid. The simulation is designed with a $8\ \mathrm{m}$ long shock tube, in order to give time for the solid medium motion to take place, and also prevent reflected waves on the outer boundaries from interacting again with the solid medium. The solid medium equations (1) are discretized with a classical (implicit) Newmark algorithm. The mesh size h is set to 1 mm, and $(C_{stab}, s_0) = (0.9, 0.585h)$. The increased cutoff scale s_0 here aims at suppressing the high frequency noise previously witnessed. This allows us to recover, in Figure 6b below, a sinusoidal shape for the horizontal displacement, which is coherent with the free theoretical response of a linear oscillator (designed with the same mechanical parameters) in pseudo-periodic regime.



(a) Zoom on the initial pressure field (2.5 vs 1 bar).



(b) Horizontal displacement

Figure 6: Shock wave - 2×2 moving array of disks.

4 CONCLUSION

This article briefly described the key ingredients of a new contribution in the wide literature of homogenization and multi-scale methods, here applied to transverse pressure waves propagating within a congested solid medium. In a will to build a self-sustained model, which can bypass the major limitations still encountered in literature, this work promotes an original use of Continuous Wavelet Transform (CWT). A two-steps process of "weak-extension" +

”weak-convolution” of the original fluid PDEs with an analysing wavelet (or scaling function) results in spatially-filtered PDEs governing an equivalent/homogenized fluid. The new variables are moreover defined as the wavelet coefficients of the original variables. More importantly, thanks to CWT and its inverse transform, the wavelet-based model possesses the brand new ability to connect resolved and unresolved scales without any *ad hoc* model, and to rigorously handle the original boundary conditions. It was also emphasized how the inverse wavelet transform can be used to explicitly compute, if necessary, nonlinear terms. Several numerical tests with steady solid obstacles allowed to assess the model capability to accurately reconstruct a 2D reference pressure field, and thus the dynamic load applied to the solid medium. A preliminary numerical test involving a 2×2 moving array of disks allowed to implement successfully a first coupling between the equivalent/homogenized fluid and an underlying rigid solid medium possessing 2 degrees of freedom. These early results shall be completed with additional testing in order to build a robust 2D (and eventually 3D) fluid-structure solver.

Finally, the framework of the wavelet-based multi-scale and homogenized model can naturally be applied to represent in a sparse way a generic continuum medium governed by conservation laws. This shall be the topic of an upcoming article.

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