30th NENE 2021 September 6 - 9 BLED-SLOVENIA
International Conference Nuclear Energy for New Europe

Advanced Modelling And Stability Analysis For Nuclear Reactors

Carolina Introini, Antonio Cammi, Stefano Lorenzi, Francesca Celsa Giacobbo
Politecnico di Milano, Department of Energy
via La Masa 34
I-20156, Milan, Italy
carolina.introini@polimi.it, antonio.cammi@polimi.it, stefano.lorenzi@polimi.it,
francesca.giacobbo@polimi.it

ABSTRACT

This work focuses on the development and improvement of advanced modelling strategies for the study of traditional and innovative nuclear reactor design. The primary aim is the integration of MOR and DA techniques to provide accurate and computationally efficient algorithms suitable for real-time and control applications. As nuclear reactors present unique features compared to conventional systems, the identification of the best performing advanced modelling strategies remains an ongoing challenge. In this sense, MOR techniques offer a promising solution to the trade-off between solution accuracy and computational times, especially in multi-query simulation scenarios. Their joint use with data-driven techniques can help improve the computational accuracy by integrating the model simulation with the experimental data efficiently. The three techniques considered and developed in this study have been tested on the TRIGA Mark II reactor, using experimental data and a CFD model of the reactor core, focusing on the prediction of the steady-state beyond the observed data (DMD), the improvement of the prediction compared to the experimental data (POD-KF), and the optimisation of the sensor positioning within the core along with the reconstruction of fields for which measurements are not available (GEIM).

1 INTRODUCTION

Because of the complexity of Generation-IV concepts, the trend of nuclear reactor analysis relies on high-fidelity 3D multi-physics modelling and simulation. However, despite the advancements in terms of computational hardware and numerical methods, one of the main challenge of high-fidelity modelling remains the high computational cost of simulations, in particular for all applications where multiple simulations of the same phenomena (for example, with different initial conditions or parameters value) must be performed quickly and efficiently. The common strategy to sidestep this issue relies either on simulating single sections of the reactor core or on adopting simpler models, whose results are then extrapolated. Inevitably a trade-off between computational cost and solution accuracy arises, which in the nuclear field it is extremely skewed towards accuracy, especially because of the strict safety requirements [1].

Model order reduction (MOR) [2] techniques represent a viable strategy to reduce the computational times whilst preserving the solution accuracy, especially in the design and parameter optimisation phases which require multiple, fast simulations. Once the reduced order model (ROM) is built starting from some available data, it becomes the optimal solution for multi-query scenarios, where multiple simulations (for example, for different values of the parameters under investigation) must be performed. ROMs can also be used during the operation phase in control-related applications, for example to quickly predict the future evolution of the system following anomalies in the operating conditions and to compute those fields of interest for which experimental data are hard to obtain (for example, in-core quantities). Still, standard ROM techniques may not be able to provide the needed accuracy for the safety requirements, and they have some limitations that must be considered when studying their applicability [4].

The primary objective of this work is to develop and investigate advanced reduced-order modelling strategies for the study of traditional and innovative nuclear reactor designs, overcoming the limitations associated with the ROMs by integrating the proposed MOR techniques with data assimilation (DA) approaches. For example, traditional MOR algorithms present only seldom integration between the available experimental data and the simulation, and once the ROM is created, it operates as a black box. Additionally, the most widespread ROM techniques need to know the underlying equations of the system under analysis, which are usually approximated as linear ones for simplicity thus introducing further uncertainties, and some of the physical phenomena occurring in the real-world system may not be known a-priori, and thus neglected [7]. Whereas this equation-based approach was enough for traditional systems, with increasing complexity the paradigm is shifting towards equation-free and DA methods.

Data assimilation techniques [3] lie between model-based and pure data-driven methods, as they use both experimental data and the information coming from the numerical model. Uncertainty, noise fluctuations and approximations heavily influence both source of information: by combining the two, the experimental data can help inform the model about the state of the real system with local information, and the model can help discriminate between instrument and stochastic uncertainty of the observation, while being able to predict the evolution of the system beyond the available measurements. The dynamic integration of data can then improve the predictive accuracy of the model and identify the locations where sensors may be more useful.

This work develops and improves three different ROM approaches for the analysis, optimisation and control of nuclear reactors, each with distinct advantages and limitations to tackle different issues and challenges. These techniques go beyond the current state-of-the-art modelling solutions for nuclear reactors by providing an adequate trade-off between computational cost and accuracy, as well considering their potential future application for the control and real-time analysis. All three approaches have been tested and validated using the developed OpenFOAM computational fluid-dynamic (CFD) model of the TRIGA Mark II research reactor (University of Pavia) as benchmark test case and the available experimental data on the coolant temperature [8]. Despite its technology, this reactor is a very good benchmark case even for Generation-IV concepts, because it has many characteristics in common (natural circulation for cooling, difficulties in performing sub-channel analysis, asymmetric core geometry).

2 METHODOLOGIES

In general, computational reduction approaches aim at reliably retain the governing dynamics of the system whilst significantly reduce the computational cost of performing multiple simulations. The fundamental assumption behind any ROM technique is that a few dominant basis can accurately describe the behaviour of the whole system [5]. The selection of the set of optimal basis modes is critical to create a lower-dimensional but accurate enough approximation of the full-order system. As such, ROM techniques relies on a computationally expensive offline stage to extract the dominant modes and build the reduced order model, based either on experimental data (data-driven ROM) or snapshots obtained by simulating the full-order model

(FOM) for a limited number of parameters (model-based ROM). In both cases, this costly step is performed only once, and the obtained, computationally cheaper reduced order model can be used in the so-called online phase for running multiple simulations, for example for additional values of the parameters for which the FOM solution is unavailable [6].

2.1 Dynamic Mode Decomposition

Traditional MOR techniques have been successfully applied to power systems to identify the governing mode shapes, and in the nuclear engineering community they have been mostly applied to study the long-term behaviour of the system. However, even traditional reactor transients have much different short-time dynamics, and Generation-IV concepts may present local, unexpected phenomena not included in the underlying model (neglecting their contribution).

Dynamic Mode Decomposition (DMD) is an equation-free MOR technique able to represent even complex models with explicit temporal dynamics based only on the observed data, without requiring any knowledge of the underlying governing equations. DMD is based on the Singular Value Decomposition (SVD), and allows the extraction of the time-varying characteristics of the system and of the governing dynamic structures from the available snapshots. In addition, compared to other MOR methodologies, DMD allows the evaluation of a low-dimensional surrogate of the dynamic matrix $\mathbf{A}$, on which dynamic and stability analysis can be performed, and to predict the future system behaviour even without observations [9].

As its application for nuclear-related applications is still not widespread, this work carries out an optimisation of the DMD algorithm for the reconstruction and prediction of reactor transients. The code was developed in MATLAB using the standard C++ language, along with routines to communicate with OpenFOAM. The major advantage of this modelling strategy is the possibility to use the code regardless of the origin of the data, using proper routines, however the main drawback is the additional time required to convert the data in the appropriate format and the limited allowed dimensions of the snapshots matrix.

2.1.1 Non-Modal Stability Analysis

Through linearisation of the system, traditional stability techniques are commonly used to assess the robustness of the system to perturbations by evaluating its asymptotic response. These techniques do not give any information regarding the behaviour of the system immediately following the perturbation; non-negligible but transient short-term amplification of the disturbance may occur even for systems with no predicted long-term growth, which may amplify minor perturbations into ones that may trigger the transition from stable to unstable [13].

This work adopts the non-modal stability method (developed for hydrodynamic problems) to the study of the stability of nuclear reactors. This method focuses on the study of the short-term dynamics, and how the system eventually reaches the asymptotic stable state, by considering the overall time response of the system to the input variables by studying the ε-pseudospectra of the dynamic matrix $\mathbf{A}$ [14]. As the DMD method computes a low-dimensional approximation of this matrix starting only from the available data, its integration with the non-modal stability method to perform stability analysis in real-time is straightforward. The code for the non-modal analysis was written in MATLAB using the EigTool library as reference.

2.2 Proper Orthogonal Decomposition with Kalman Filtering

Proper Orthogonal Decomposition (POD) is one of the most widespread model-based MOR methodologies for fluid-dynamics applications. It is based on solving the FOM for only a selected number of instances of the parameters; these FOM solutions are then used to construct the ROM [15]. Based on this computationally demanding offline step, performed only once, many computationally inexpensive online simulations for different instances of the parameters can be performed. The need of having high-fidelity PDE solutions from which to build the ROM represents the bottleneck of this method and, being based on pre-computed solutions of the FOM, it lacks predictive capabilities: to include up-to-date information, new full-order snapshots must be created, and the ROM must be rebuilt, performing again the expensive offline stage. Another drawback is related to the fact that the reduced POD space is built based on a maximum energy criterion, leaving out the less energetic modes which, for turbulent flows, are the most dissipative and thus contain local information about the state of the system.

This work proposes a viable way to improve the spatial and predictive capabilities of the POD technique by enriching the ROM model using additional information coming from experimental data: this is done by integrating the Kalman filter (KF) [16], a DA algorithm, with the POD reduction techniques (POD-KF). Compared to literature [17], where the filtering procedure is applied on the projection of the ROM variables during the online stage and thus the KF refers to the FOM, this work proposes a novel strategy for the integration by implementing the DA algorithm directly in the offline stage during the construction of the ROM. This way, the KF refers to the reduced space, limiting the additional computational cost, and the ROM itself now includes also the information coming from the experimental data. The code was implemented using the ITHACA-FV and Eigen C++ library for OpenFOAM [10].

2.3 Generalised Empirical Interpolation Method

As they rely on experimental data, DA techniques are limited by the availability of the measurements and by the number of available sensors. Measured data are often sparse in the physical domain, and the direct observation of some quantities may be hard to the point that their experimental investigation is not possible. An appropriate sensor positioning strategy must then be defined to extract as much information as possible from the system, even with sparse measurement locations and unobserved variables, to reconstruct the entire state through observations taken only on some fields of interest.

Among all reduction order techniques, the Generalised Empirical Interpolation Method (GEIM) [11] allow to tackle at the same time both the problem of optimal sensor positioning and the reconstruction of the state of the system in real-time. The GEIM is an equation-free method, as it does not require any modelling of the underlying equations, based on a greedy procedure to extract both the basis functions and a set of interpolating points (representing the optimal location of the sensors) from the available data (which can be both full-order snapshots of the system and experimental data); these information are then used both to solve the problem of sensor positioning and of real-time reconstruction, both direct and indirect.

A situation commonly encountered in thermal-hydraulics and nuclear systems is the presence of temperature sensors and the lack of ones for velocity or mass flow rate [18]. The GEIM algorithm has been improved to use these temperature data to reconstruct the velocity field having no kind of information on it, under the strong assumption that the reduced interpolation coefficients for temperature are equal to that for velocity. The algorithm has been tweaked in order to use sensors in fixed locations, to better represent the presence of sensors in fixed and sparse locations of the domain. The GEIM algorithm and its modification for the indirect interpolation of fields have been implemented directly into OpenFOAM: this choice makes the code more portable but less optimised, and the code now depends on the OpenFOAM version with which the FOM snapshots are computed (or the software with which the data are acquired).

3 RESULTS AND DISCUSSION

3.1 Dynamic Mode Decomposition

Following a preliminary test on the Molten Salt Fast Reactor (MSFR) [12], the developed DMD algorithm was used to study the heating transient and nominal power operation of the TRIGA Mark II reactor core. The ROM was build using high-fidelity snapshots obtained from CFD simulations, and a sensitivity analysis over the sampling time and the truncation rank (the dimension of the ROM) was also carried out to identify the optimal model parameters.

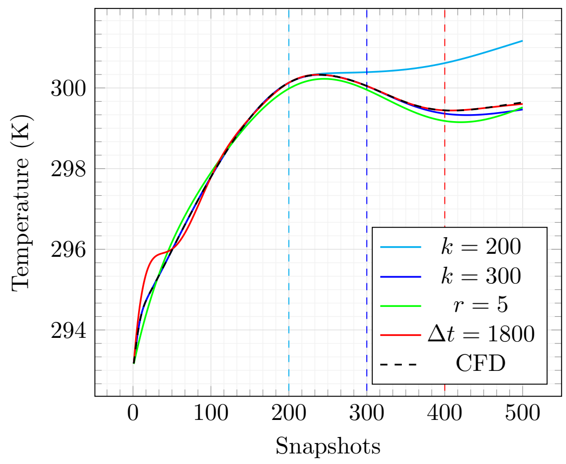

Figure 1: Predicted average core temperature for different values of the ROM parameters (final sampling time, truncation rank, sampling time). The standard parameters value are $k = 400$, $r = 25$ and $\Delta t = 360$ s. The vertical lines represent the last snapshot used to build the DMD model.

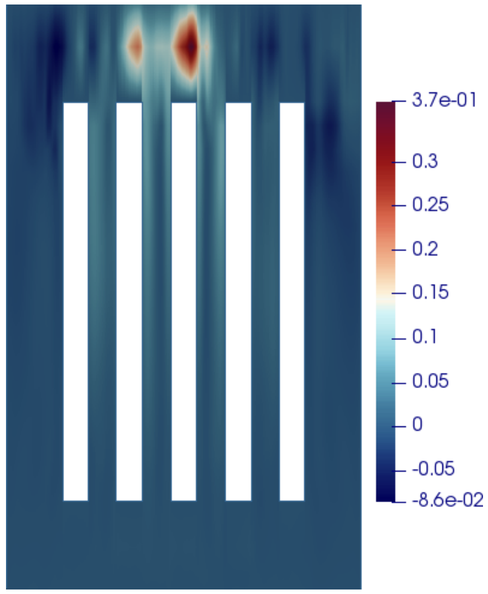

Figure 2: Absolute misfit (computed as the difference between the CFD value and the DMD predicted one) at the end of the transient for the magnitude of the velocity field ($k = 300$, $r = 25$, $\Delta t = 360$).

Figure 1 shows the average core temperature computed with the DMD algorithm for different values of the model parameters (index of the last snapshot, ROM size, sampling time). In general, for an accurate reconstruction the information must be available especially at the start of the transient, when the system is still evolving. Once a pseudo steady-state is reached, the DMD algorithm can capture it based only on the past evolution of the system. The truncation rank and the sampling time have less effect on the reconstruction than the index of the last snapshot, although the sampling time must be fine enough to capture all the transient dynamics. The most significant result is that, in principle, the FOM data acquisition or the experimental campaign could be stopped early, as the previously computed time steps are enough to propagate the state of the system with reduced computational effort, especially the closer the cutoff value is to the steady-state. Figure 2 shows the absolute misfit between the core velocity reconstruction in the XY mid-plane as predicted by the DMD and the one obtained by the CFD simulation. The DMD can predict the evolution of velocity with reasonable accuracy: higher errors are found in the upper core region where recirculation from and to the upper pool is expected, and where the state of the system is farther from the steady-state.

3.1.1 Non-Modal Stability Analysis

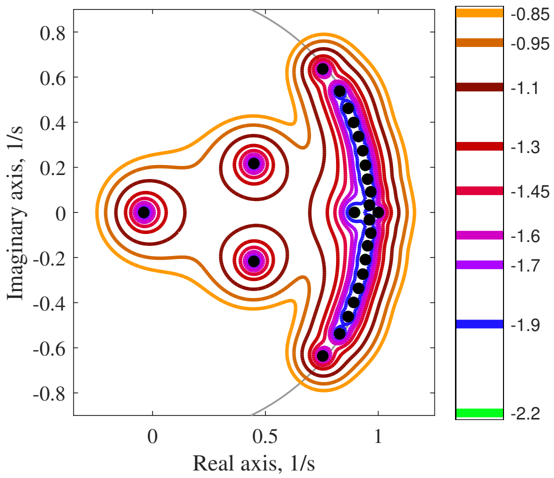

Figure 3: Pseudo-spectra of the discrete-time dynamic matrix $\mathbf{A}$ for different value of the perturbation. The black dots represents the first 25 discrete eigenvalues computed by the DMD algorithm. The colour bar shows the norm of the perturbation on a logarithmic scale, expressed as $\log \varepsilon$, with ε equal to the magnitude of the perturbation.

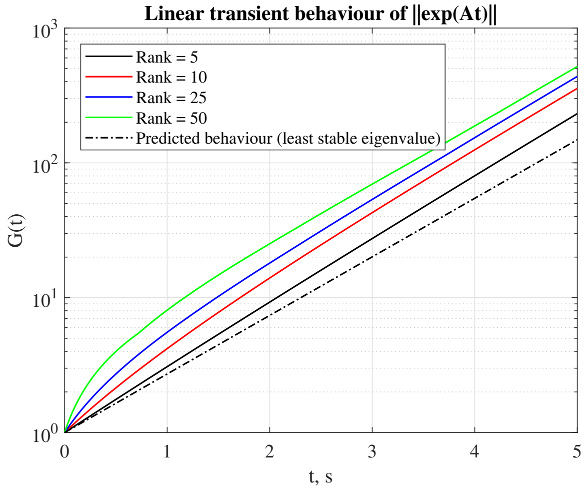

Figure 4: Logarithmic transient growth of the perturbation energy $G(t)$ (maximum energy amplification over all possible initial conditions) as computed by non-modal stability analysis for different values of the truncation rank and the least stable eigenvalues.

Figure 3 shows the ε-pseudospectra for the first 25 discrete thermal-hydraulics natural circulation eigenvalues of the approximation matrix $\tilde{\mathbf{A}}$ computed by the DMD model. These eigenvalues predict an unstable system also for linear stability theory, and non-modal stability confirms this behaviour (the contour levels extend beyond the unit circle). Figure 4 shows the behaviour of the maximum energy amplification as predicted by non-modal stability theory (for different values of the truncation rank r and its comparison with the one predicted by the least stable eigenvalues. Unbounded growth occurs in both cases, but linear stability underestimates this growth as it does not consider the contribution of the stable eigenvalues whose pseudo-spectrum extends beyond the unit circle.

3.2 Proper-Orthogonal Decomposition

The model was first assessed with respect to the classic CFD benchmark of the backward facing step, considering a parametrised MOR problem taking as parameter the step inlet velocity (not reported here for sake of brevity). Following this validation, the POD-KF algorithm was applied to an instrumented cooling channel of the TRIGA Mark II reactor, equipped with a measurement rod with eight equally spaced sensors, using the available experimental data.

Figure 5 shows the locally reconstructed temperature in position 1 (corresponding to the channel inlet) and 5 of the measurement rod, using both the standard POD and the integrated POD-KF (with the same dimension of the reduced order space). The improvement in the reconstruction is clear: the results obtained with the POD-KF are closer to the experimental ones, and the POD-KF is able to predict the temperature increase in the lower, adiabatic zone (which, conversely, was not observed by the standard POD). Figure 6 shows the misfit between FOM and reconstructed one for both algorithms. Despite the lack of experimental data on velocity,

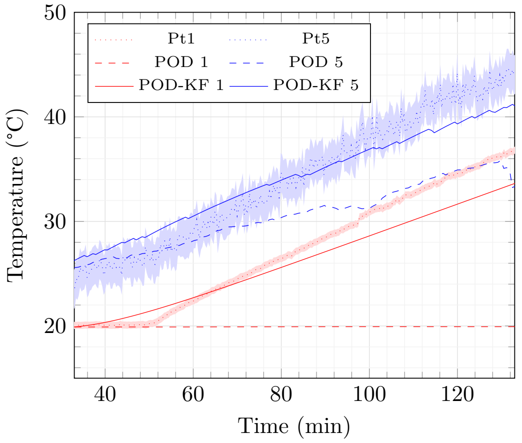

Figure 5: Temperature reconstruction with the standard POD (green) and the POD-KF in position 1 (inlet) and 5 (0.4 cm from the inlet) and comparison with the recorded experimental data using 30 basis and sampling frequency of 30 seconds.

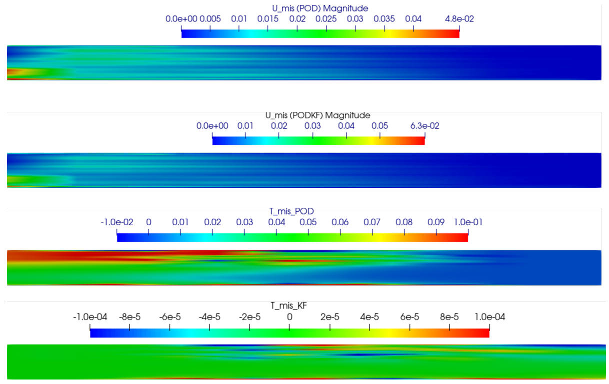

Figure 6: Velocity and temperature misfit with respect to the FOM for the standard POD (top) and the filtered one (bottom), in both cases using 30 basis and sampling frequency of 30 seconds.

some improvement in the reconstruction is observed within the whole channel, and not only along the measurement rod.

3.3 Generalised Empirical Interpolation Method

Following its application on the single instrumented channel in order to validate the method and select the optimal value of the algorithm parameters, the GEIM was applied to the study of the TRIGA Mark II reactor core, using pre-processed experimental data and FOM snapshots to assess the performance of the method in reconstructing the unobserved velocity field and to identify the optimal positioning of the measurement rods.

Figure 7 shows the average relative L^2 reconstruction error obtained with the GEIM approach for temperature and velocity, using only sensors of the former. This error is compared to the POD one, taken as reference ROM.Acceptable convergence is obtained with the GEIM method, as 32 sensors are enough to bring the reconstruction error for temperature around $1E-4$. Overall, the optimised GEIM (with optimised sensor locations) performs better than the fixed GEIM for temperature, and slightly worse for velocity (this means that the optimal positioning for velocity is different than the one for temperature). Globally, the POD-KF has better accuracy but longer computational times and, most importantly, it can reconstruct only fields for which FOM data are available. On the contrary, with the GEIM remarkable results are obtained for the reconstrunction of the velocity field (Figure 8): with no data on it, the interpolation with the stabilised GEIM method can get a reconstruction error around $1E-2$. This result is especially relevant for the analysis of nuclear reactors because it means that, in principle, fields of interest may be estimated with reasonable accuracy without the need of in-core sensors, thus avoiding all the issues related to the performance and maintenance of sensors in a highly radioactive environment.

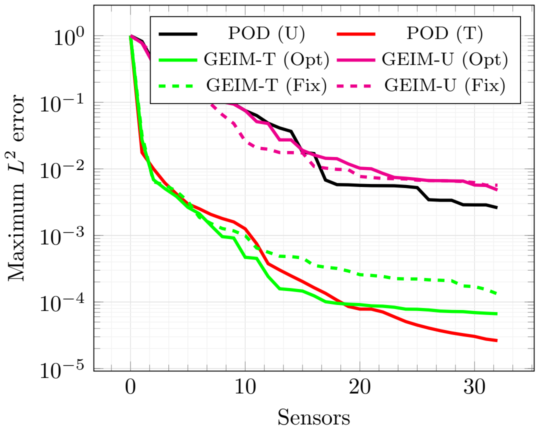

Figure 7: Average relative L^2 reconstruction error for the POD (taken as reference, black line), the GEIM for temperature (with sensors, red line), the GEIM for velocity (without sensors, using only the information on temperature, green line), and the GEIM following optimisation of the sensor positions (dashed) as a function of the number of available sensors.

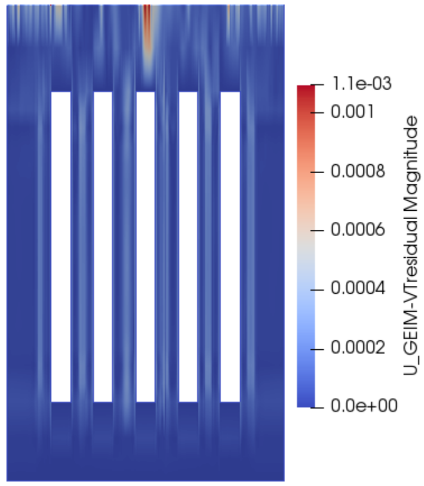

Figure 8: Misfit between the FOM velocity and the reconstructed one using the GEIM (with sensor optimisation) in the XY mid-plane at the end of the transient, using fixed sensor locations.

4 CONCLUSIONS

This work investigates and develops different MOR and DA techniques for the analysis, optimisation and control of nuclear reactor systems. The presented techniques go beyond the current state-of-the-art modelling solutions for nuclear reactors, by providing an adequate trade-off between computational cost and accuracy, as well considering their potential future application for the control and real-time analysis of the actual system. Three different MOR techniques have been considered and tested against the TRIGA Mark II research reactor using both experimental data and a CFD model.

The DMD method was optimised for the analysis of nuclear reactor core, and the results show how this technique is the best one to investigate the underlying dynamics of the system and when stability analyses (and thus a surrogate of the dynamic matrix) are required (using, in this case, the non-modal stability method). The DMD can extract the dominant dynamical structures to extend the model prediction beyond the last available snapshot, especially when dense observations are available In terms of computational time, the DMD is the fastest method in terms of online reconstruct, however in its current implementation it is severely penalised by the time required to convert the data from the native format to the MATLAB one.

The POD method was integrated with the KF to build a hybrid ROM based both on the underlying model and on the available experimental data. This method is the best one when dealing with parametrised problems, provided that the FOM is not too big in terms of space discretisation. Among the three MOR algorithms, the POD-KF is the computationally most expensive one, making it the less suitable for real-time applications.

The GEIM, a data-driven equation free-method, was used to tackle the optimisation of sensor positioning within the TRIGA reactor core and to assess the possibility to reconstruct the velocity field using only temperature sensors. The method proved to be able to predict the velocity field with reasonable accuracy even without any a-priori information on it and in presence of noisy observations, making it the best solution when the location of the sensors must be optimised and when information on some fields of interest is not available. Its computational cost is slightly higher than the DMD.

Future developments of the presented work involve the application of the developed methodologies to innovative nuclear reactor concepts, mainly the (MSFR). Further validation will be carried out using the DYNASTY facility available at Politecnico di Milano. Further improvements of the presented techniques are also envisaged using the last advancements in MOR techniques, along with the possibility to use Neural Networks (NN) as interpolation and reduction method. The performance of the proposed techniques will also be investigated with respect to multi-physics problems, for example using burnup data from reactor fuel or neutron fluxes. As multi-physics (MP) modelling is becoming the state-of-the-art solution for innovative nuclear reactor designs, their coupling with MOR techniques is of interest to overcome the inherent limitations of full-order MP approaches, namely the prohibitive computational costs and model complexity. In this sense, it is worth investigating whether the use of ROM methodologies could envisage a novel approach to MP problems, by performing a coupling of the various physics at the reduced-order level.

REFERENCES

[1] US DOE, “Generation IV International Forum: A Technology Road-Map For Generation IV Nuclear Systems”, Technical Report, 2002

[2] W. Schilders et al., Model Order Reduction: Theory, Research Aspects and Applications, Springer-Verlag, Berlin (GER), 2008.

[3] F. Darema, “Dynamic Data Driven Applications Systems: New Capabilities for Application Simulations and Measurements”, In: Computational Science - ICCS 2005: Lecture Notes in Computer Science, Vol. 3515, Springer, Berlin (GER), 2005

[4] G. Rozza, M. H. Malik, N. Demo et al., “Advances in reduced order methods for parametric industrial problems in computational fluid dynamics”, Proceedings of the 6th European Conference on Computational Mechanics: Solids, Structures and Coupled Problems, ECCM 2018 and 7th European Conference on Computational Fluid Dynamics, ECFD 2018, 2020

[5] A. Quarteroni et al., Reduced Order Methods for Modelling and Computational Reduction, Springer-Verlag, Berlin (GER), 2004.

[6] S. Brunton, N. Kutz, Data Driven Science & Engineering - Machine Learning, Dynamical System and Control, Cambridge University Press (UK), 2019.

[7] M. Asch, M. Boquet, N. Nodet, Data Assimilation: Methods, Algorithms, and Applications, Cambridge University Press (UK), 2017

[8] C. Introini, A. Cammi, S. Lorenzi et al., “Complete Thermal-Hydraulic Modelling of the Pavia TRIGA Mark II Research Reactor for the Study of the Natural Circulation Regime ”, Proceedings of the 12th International Topical Meeting on Nuclear Reactor Thermal-Hydraulics, Operation and Safety (NUTHOS-12), 2018

[9] N. Kutz, Data-Driven Modelling and Scientific Computation: Methods for Complex Systems and Big Data, Oxford University Press (UK), 2013

[10] G. Stabile, G. Rozza et al., "Finite volume POD-Galerkin stabilised ROM for the parametrized Navier-Stokes equations", Computers & Fluids 173 (15), 2018, pp. 273–284

[11] Y. Maday et al., "The GEIM: stability theory on Hilbert spaces with an application of the Stokes equations", Computer Methods in Applied Mechanics and Engineering, 287, 2015, pp. 310—334

[12] A. Di Ronco, C. Introini, A. Cammi et al., "Dynamic Mode Decomposition for the Stability Analysis of the MSFR Core", Nuclear Engineering and Design, 362, 2020, no. 110529

[13] P. Schmidt, "Non-modal Stability Theory", Annual Review of Fluid Mechanics, 39, 2007, pp. 129–162

[14] L. Trefethen, Spectra and Pseudo-spectra: the Behaviour of Non-normal Matrices and Operators, Princeton University Press (USA), 2005

[15] A. Manzoni, A. Quarteroni, G. Rozza, "Computational Reduction for Parametrized PDEs: Strategies and Applications", Milan Journal of Mathematics, 80, 2021, pp. 283-–309

[16] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems", Journal of Basic Engineering, 82(1), 1960, pp. 35–45

[17] S. Lorenzi, A. Cammi, G. Rozza et al., "Reduced order methods for the improvement of control-oriented modelling of NPPs", Proceedings of the 10th International Topical Meeting on Nuclear Plant Instrumentation, Control, and Human-Machine Interface Technologies (NPIC& HMIT), 2017

[18] S. Cavalleri, "Non-intrusive reduced order methods for measurement selection and state estimation in presence of noise", Master Thesis, Politecnico di Milano, 2002